Baryon Properties in Soliton Models *

Norberto N. Scoccola † **

Abstract. We report on the results of some calculations of the radiative decays of decuplet baryons in the context of topological chiral soliton models. Such results are compared with those of alternative baryon models.

1 Introduction

At present, only few data are available on the electromagnetic decays of the \( \frac{3}{2}^+ \) baryons. Although recently the reaction \( \Delta \rightarrow N\gamma \) has carefully been analyzed at MAMI, the decay parameters are still unknown for those \( J = \frac{3}{2} \) to \( J = \frac{1}{2} \) transitions which involve strange baryons. Upcoming experiments at JLAB [1] are expected to provide some data on these radiative decays soon and thus give more insight in the pattern of flavor symmetry breaking. Theoretical studies of these decays have been performed in a number of quark-based models like i.g. the non-relativistic quark model [2, 3], a quenched lattice calculation [3], etc. Here, we complete those studies by presenting the predictions of the Skyrme-type soliton models. In these models, baryons emerge as solitons configurations of the pseudoscalar mesons. The corresponding non-linear chiral action describes the main features of QCD at low energies. Since we are interested in hyperon properties, and in order to account for the non-negligible strange quark mass, convenient chiral symmetry breaking terms have to be included. One way to treat the kaon fields in the presence of these symmetry breaking terms is to assume that they correspond to small amplitude fluctuations off the soliton. In this picture, called "bound state approach" (BSA), hyperons appear as kaon-soliton bound states[4]. Alternatively, the "rigid rotator approach" (RRA) starts from a flavor symmetric formulation wherein non-vanishing kaon fields arise from a rigid rotation of the classical pion field. The associated collective coordinates, which parametrize these large amplitude fluctuations off the soliton, are canonically quantized to generate states which

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* Work supported in part by a grant of Fundación Antorchas.
† Fellow of the CONICET, Argentina.
** E-mail address: scoccola@tandar.cnea.edu.ar
possess the quantum numbers of physical hyperons. It turns out that the resulting collective Hamiltonian can be diagonalized exactly even in the presence of flavor symmetry breaking [5]. In this contribution we will basically concentrate on this second method although results for the BSA will be also presented.

2 The model

Our starting point is a gauged effective chiral action with appropriate symmetry breaking terms written in terms of pseudoscalar octet $\phi$ and the photon field $A_\mu$. The former is non–linearly represented by the chiral field $U = \exp(i\phi)$. Such action reads

$$\Gamma = \int d^4x \left\{ \frac{f_\pi^2}{4} \text{Tr} \left[ D_\mu U (D^\mu U) \right] + \frac{1}{32\epsilon^2} \text{Tr} \left[ \left( U^\dagger D_\mu U, U^\dagger D_\nu U \right)^2 \right] \right\} + \Gamma_{an} + \Gamma_{sb}. \quad (1)$$

Here, $f_\pi = 93$ MeV is the pion decay constant and $\epsilon$ is the Skyrme parameter. The covariant derivative $D_\mu U = \partial_\mu U + i e A_\mu [Q, U]$ is defined via the usual electric charge matrix $Q$. Moreover, $\Gamma_{an}$ is the Wess-Zumino action gauged to contain the photon field and $\Gamma_{sb}$ a suitable symmetry breaking term that accounts for the finite pion and kaon masses as well as for the difference between the kaon and pion decay constants. The explicit expressions of $\Gamma_{an}$ and $\Gamma_{sb}$ in terms of $U$ and the electromagnetic field can be found in i.e. Ref.[6].

To generate baryon states of good spin and flavor quantum numbers we consider the $SU(3)$–rotating hedgehog configuration,

$$U(r, t) = A(t) \exp \left[ i \tau \cdot \hat{r} F(r) \right] A(t)^\dagger. \quad (2)$$

Inserting this ansatz and following the standard canonical quantization one obtains the collective Hamiltonian

$$H = M_{sot} + \left( \frac{1}{2\Theta_\pi^2} - \frac{1}{2\Theta_K^2} \right) J^2 + \frac{1}{2\Theta_K^2} C_2 [SU(3)] - \frac{3}{8\Theta_K^2} + \Phi \left( 1 - D_{88} \right). \quad (3)$$

In Eq. (3), $\Theta_\pi$ and $\Theta_K$ are the moments of inertia in the pionic and kaonic directions, respectively and $\Phi$ the symmetry breaking strength (see Ref.[7] for details). $J$ denotes the spin operator while $C_2 [SU(3)]$ refers to the quadratic Casimir operator of $SU(3)$. This collective Hamiltonian can be diagonalized exactly. The eigenfunctions $\Psi_B (A) = \langle A | B \rangle$ of the collective Hamiltonian (3) are identified as the wave–functions corresponding to baryon $B$. These are distorted $SU(3)$ D–functions since the resulting baryon eigenstates contain sizable admixtures of baryon states with appropriate spin and flavor quantum numbers in higher dimensional representations of $SU(3)$.

For our numerical calculations we take the meson masses and decay constants at their physical values and adjust the Skyrme parameter to $\epsilon = 3.9$. The observed mass differences for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons are quite accurately reproduced for this set of values[7]. On the other hand, the absolute values of the baryons masses turn out to be all too large. Recently, it has been shown
that even within the SU(3) Skyrme model this problem can be cured when quantum corrections of \(O(N_C^0)\) are included [8]. As we will see below these quantum corrections might also play an important role in the hyperon decay amplitudes.

3 Electromagnetic Decay Widths

For the radiative decays of the decuplet hyperons we are interested in both \(M1\) and \(E2\) allowed transitions. The corresponding partial widths can be expressed in terms of the baryon e.m. current \(J^\mu_m\) as

\[
\Gamma_{E2} = \frac{675}{8} \varepsilon^2 q \left| \langle \Psi_{J=\frac{3}{2}} \left[ \int d^3 r j_2(qr) \left( \frac{z^2}{r^2} - \frac{1}{3} \right) J^{em}_0 | \Psi_{J=\frac{3}{2}} \right] \right|^2, \tag{4}
\]

\[
\Gamma_{M1} = 18 \varepsilon^2 q \left| \langle \Psi_{J=\frac{3}{2}} \left[ \frac{1}{2} \int d^3 r j_1(qr) \epsilon_{3ij} \tilde{r}_i J^{em}_j | \Psi_{J=\frac{3}{2}} \right] \right|^2. \tag{5}
\]

where \(q\) is the photon momentum and \(j_i(qr)\) are the spherical Bessel functions.

In our model, the general form of \(J^\mu_m\) can be directly obtained from the gauged effective action (Eq. (1)). Using the ansatz (Eq. (2)), one can then get the explicit expression of the integrands in Eqs. (4,5). These expressions can be found in Ref.[6]. It should be noticed that in deriving Eq.(4) some approximations according to the Siegert’s theorem have been made. The impact of such approximations on the \(E2 - \Delta N\) transition has been recently studied in the two flavor reduction of the model[9]. It turns out that the kinematical corrections coming from the low-momentum expansion of the Bessel functions are of the order of 5%. In addition to this, pion fluctuations off the rotating soliton have to be included to consistently satisfy the continuity equation \(\partial_\mu J^\mu = 0\) at sub-leading order in \(1/N_C\). These induced fields account for shortcomings in the collective quantization and give corrections of the order of (\(\sim 25\%\)) for the \(\Delta N\) case. Unfortunately such an inclusion of induced fields seems to be unfeasible in the three flavor model with symmetry breaking included. In any case, since the decuplet decays are almost completely \(M1\)-dominated these uncertainties do not affect the total decay widths in any significant way.

Results for the total decay widths are given in Table 1. There we list the corresponding widths normalized to the \(\Delta N\) value. As in the case of magnetic moments, the calculated \(\Delta N\) transition amplitude turns to be roughly 30% smaller than the empirical value. It has been recently shown, however, that the inclusion of next-to-leading order quantum corrections not only solves nucleon mass problem as already mentioned, but also that of the magnetic moments[10]. Preliminary results indicate that satisfactory \(\Delta N\) decay amplitudes can be obtained in that case. From Table 1 we observe that the predictions of the present model (RRA) are very similar to those of the alternative approach to SU(3) Skyrme model (BSA) and also to those of quark-based models. In particular, in all models the \(U-\)spin selection rules that predict small \(\Sigma^{*-} \rightarrow \gamma \Sigma^-\) and \(\Xi^{*-} \rightarrow \gamma \Xi^-\) are rather well satisfied.
Table 1. Total decay widths normalized to that of the $\Delta \rightarrow \gamma N$ transition in various models. The results for RRA correspond to the model describe in the text while the BSA results have been taken from Ref.[11]. The data for the quark model (QM) and lattice calculation (Lat) are taken from Refs.[2, 3].

<table>
<thead>
<tr>
<th>Transition</th>
<th>RRA</th>
<th>BSA</th>
<th>QM</th>
<th>Lat.</th>
</tr>
</thead>
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<tr>
<td>$\Sigma^{*0} \rightarrow \gamma\Lambda$</td>
<td>0.653</td>
<td>0.765</td>
<td>-</td>
<td>0.703</td>
</tr>
<tr>
<td>$\Sigma^{*-} \rightarrow \gamma\Sigma^-$</td>
<td>0.007</td>
<td>0.010</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>$\Sigma^{*0} \rightarrow \gamma\Sigma^0$</td>
<td>0.035</td>
<td>0.037</td>
<td>0.040</td>
<td>0.055</td>
</tr>
<tr>
<td>$\Sigma^{*+} \rightarrow \gamma\Sigma^+$</td>
<td>0.210</td>
<td>0.233</td>
<td>0.233</td>
<td>0.303</td>
</tr>
<tr>
<td>$\Xi^{*-} \rightarrow \gamma\Xi^-$</td>
<td>0.011</td>
<td>0.039</td>
<td>0.009</td>
<td>0.012</td>
</tr>
<tr>
<td>$\Xi^{*0} \rightarrow \gamma\Xi^0$</td>
<td>0.313</td>
<td>0.412</td>
<td>0.300</td>
<td>0.415</td>
</tr>
</tbody>
</table>

4 Conclusions

In this contribution we have discussed the hyperon radiative decay in the context of the $SU(3)$ soliton models. We have found that, when normalized to that of the $\Delta N$ transition, the corresponding predictions are qualitatively similar to those of alternative quark based models.

References

1. R.A. Schumacher: Contribution to this workshop.