BARYON–BARYON INTERACTIONS
IN AN $SU(3)$ SKYRME MODEL$^a$

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ABSTRACT
We present a formalism to study baryon–baryon interactions in the framework
of the bound–state approach to the $SU(3)$ Skyrme model. The product ansatz for
the $B = 2$ system is used in a symmetrized lagrangian density. In the static approx-
imation we derive the central, spin–spin and tensor contributions to the interaction
potential.

1 - Introduction

In recent years the $SU(2)$ soliton Skyrme model$^1$ has been extended in
various ways so as to describe hyperons$^2,3$. In one of these extensions, the
bound state approach$^3$, hyperons are viewed as soliton kaon bound–states.
The one particle chiral field is expressed as

$$U_i = \sqrt{U_{\pi i} U_{K i}} \sqrt{U_{\pi i}},$$  \hspace{1cm} (1)

with

$$U_{\pi i} = \begin{pmatrix} u_i & 0 \\ 0 & 1 \end{pmatrix} \in SU(3), \hspace{1cm} u_i = e^{i\tau_i \cdot F(\tau_i)}$$

$$U_{K i} = e^{i\kappa_i / f_\kappa} \in SU(3), \hspace{0.5cm} \kappa_i = \begin{pmatrix} 0 & K_i \\ K_i^\dagger & 0 \end{pmatrix}$$

The model provides satisfactory results for masses and electromagnetic
properties$^4$ of low lying hyperons and has also been used to study strange
dibaryons systems$^5$.

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In this work we refer to the baryon-baryon interaction in the bound-state approach.

2 – The Lagrangian

The starting effective action for the two-baryon system is the one for the Skyrme model supplemented by the Wess Zumino contribution plus appropriate symmetry breaking components. The leading contributions are taken into account with the expansion of the \( U_K \)-fields to second order in \( K \).

Focusing on the quadratic term \( (\mathcal{L}_2) \) of the Skyrme lagrangian density as a characteristic one we see that it separates in single particle and interaction contributions. The product ansatz for the \( U \) field would split the interaction contribution into three pieces. One is a direct term of order \( N_G \), originated purely from the \( u_\pi \) \([SU(2)]\) fields and the other two are \( N_G^0 \) contributions in which the \( K \) fields are present in direct and exchange arrangements, i.e.,

\[
\mathcal{L}_{2,\text{int}} = \mathcal{L}_2^{SU(2)} + \mathcal{L}_2^{\text{dir}} + \mathcal{L}_2^{\text{exc}}.
\]

For instance in the \( \Sigma N \) interaction case, the \( SU(2) \) component of \( \mathcal{L}_2 \) may be written as

\[
\mathcal{L}_2^{SU(2)} = \frac{f^2}{18} \vec{r}_1 \cdot \vec{T} \mathcal{E} \left\{ (\alpha_1 \alpha_2 - \beta_1 \beta_2) \vec{\sigma}^N \cdot \vec{\sigma}^E + \gamma_1 \alpha_2 \vec{\sigma}^N \cdot \vec{r}_1 \vec{\sigma}^E \cdot \vec{r}_1 + \gamma_2 \alpha_1 \vec{\sigma}^N \cdot \vec{r}_2 \vec{\sigma}^E \cdot \vec{r}_2 + \gamma_1 \gamma_2 \vec{r}_1 \cdot \vec{r}_2 \vec{\sigma}^N \cdot \vec{r}_1 \vec{\sigma}^E \cdot \vec{r}_2 + \beta_1 \beta_2 \vec{\sigma}^N \cdot \vec{r}_2 \vec{\sigma}^E \cdot \vec{r}_1 \right\},
\]

where \( \vec{T} \mathcal{E} = 2 \vec{r} \mathcal{E} \), and

\[
\alpha = \frac{\sin 2F}{2r}, \quad \beta = \frac{\sin^2 F}{r}, \quad \gamma = F' - \alpha.
\]

Notice that \( \mathcal{L}_2^{SU(2)} \) does not contribute to the \( \Lambda N \) interaction.

3 – The Potentials

Transforming the individual variables \( \vec{r}_1 \) and \( \vec{r}_2 \) of the baryons to center of mass (\( \vec{R} \)) and interparticle (\( \vec{r} \)) coordinates, we can write the interaction potential as a function of \( \vec{r} \).

The \( \mathcal{L}_2 \) lagrangian pieces lead to

\[
V_2^{\text{dir}} = -\int d^3 R \langle H_1' H_2' | \mathcal{L}_2 | H_1 H_2 \rangle
\]
\[
\int d^3 R \left< H'_2 H'_1 | \mathcal{L}_2 | H_1 H_2 \right>,
\]
where \( |H_i\rangle \) represents the \( i \)-th baryon state.

For the \( \Sigma N \) interaction quoted above, the \( N_C \) contribution generates a potential

\[
V_{2}^{SU(2)} = \left( V_{2, S}^{SU(2)} \sigma^\Sigma \cdot \sigma^N + V_{2, T}^{SU(2)} S^\Sigma N \right) \tau^N \cdot \tau^\Sigma,
\]
with \( S^\Sigma N = 3 \sigma^\Sigma \cdot \tau \sigma^N \cdot \tau - \sigma^\Sigma \cdot \sigma^N \).

Observe that the matrix elements presented in eqs. (5) and (6) concern rotated lagrangian pieces obtained under the field transformations

\[
u_1 \rightarrow a u_1 a^\dagger, \quad u_2 \rightarrow b u_2 b^\dagger, \quad K_1 \rightarrow a K_1, \quad K_2 \rightarrow b K_2,
\]
with \( a \) and \( b \) being rotation matrices.

For the nucleon states \( |N\rangle = |I, I_z, S_z\rangle \), we have taken rotor wave functions

\[
\Psi_{I, S_z}^{(I)} = (-1)^{I+I_z} \varphi^{(I)} \sqrt{\frac{2I+1}{2\pi^2} D_{-I_z, S_z}^{(I)}} \varphi^{(I)} = (i)^{2I},
\]
where \( D_{-I_z, S_z}^{(I)} \) is an element of rotation matrix.

For the hyperon wave functions, concerning states \( |H\rangle = |I, I_z, J, J_z\rangle \), we used

\[
\Psi_{I, J_z}^{(I, J)} = \sum_{\Lambda_z} \left< J, J_z | I, J_z - \Lambda_z, \Lambda, \Lambda_z \right> \chi_{I, J_z - \Lambda_z}^{(I)} k_{\Lambda}(\tau) Y_{\Lambda} Y_{\Lambda_z},
\]
where \( \chi \) is the rotor wave function

\[
\chi_{I, J_z - \Lambda_z}^{(I)} = (-1)^{I+I_z} \varphi^{(I)} \sqrt{\frac{2I+1}{2\pi^2} D_{-I_z, J_z - \Lambda_z}^{(I)}},
\]
\( k_{\Lambda}(\tau) \) is the radial function of the bound kaon, and \( Y_{\Lambda} Y_{\Lambda_z} \) is the solid spherical harmonic

\[
Y_{\Lambda} Y_{\Lambda_z} = \begin{pmatrix} (\Lambda, \Lambda_z | l, \Lambda_z - \frac{1}{2}, \frac{1}{2}, +\frac{1}{2} \rangle Y_{l, \Lambda_z - \frac{1}{2}} \\ (\Lambda, \Lambda_z | l, \Lambda_z + \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle Y_{l, \Lambda_z + \frac{1}{2}} ) \end{pmatrix}
\]
If we deal with the ΛN interaction several simplifications occur. Namely, only exchange contributions are present in \( \mathcal{L}_2 \). The resulting potential separates in a central, a spin–spin and a tensor terms, i.e.,

\[
V_2^{exc} = V_C^{exc} + V_{SS}^{exc} \vec{\sigma}^\Lambda \cdot \vec{\sigma}^N + V_T^{exc} S^{\Lambda N},
\]

where \( \vec{\sigma}^\Lambda \) and \( \vec{\sigma}^N \) are the particle operators obtained under the correspondences \( \vec{\sigma}^\Lambda \to <\Lambda'|\vec{\sigma}|\Lambda> \) and \( \vec{\sigma}^N \to <N'|\vec{\sigma}|N> \), and \( S^{\Lambda N} \) is the usual tensor operator under the same rules.

Similar techniques lead to the construction of the different components of the potential. A systematic numerical study of these components is under way.

4 – References


(2) For the extension of \( SU(2) \) hedgehog to \( SU(3) \), \( U(\vec{r}) = e^{i \sum_{i=1,3} \vec{\lambda}_i \cdot \vec{r} F(r)} \), see, for instance, H. Yabu and K. Ando, Nucl. Phys. B301 (1988) 601;

