ELECTROMAGNETIC DECAY OF THE GIANT QUADRUPOLE RESONANCES

(1). Reaction mechanism and angular distributions of the emitted photons

B.F. BAYMAN

School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

D.R. BES, P. CURUTCHET, O. DRAGÚN, N.N. SCOCCOLA and J.E. TESTONI

Departamento de Física, Comisión Nacional de Energía Atómica, Avda. del Libertador 8250, (1429) Buenos Aires, Argentina

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Abstract: A study is made of the electromagnetic decay of the $^{208}$Pb giant quadrupole resonance excited by inelastic scattering of $^{17}$O at 380 MeV. Due to the instability of the giant quadrupole resonance with respect to compound-nucleus formation and to neutron emission, the time over which the radiation is produced is not much greater than the time in which the direct reaction occurs. Angular distributions of the $\gamma$-rays produced in transitions to several low-lying states of the residual nucleus are calculated and their dependence on effects due to the proximity of the interacting nuclei is discussed.

1. Introduction

The photon decay of the giant isoscalar quadrupole resonance (GQR) in $^{208}$Pb has recently been measured$^1$). This resonance is excited by inelastic scattering of $^{17}$O at 380 MeV. The experiment gives information on the electromagnetic branching ratios of the GQR which can be used to relate the wave function of this resonance to those of the lower-lying states in $^{208}$Pb.

In direct-reaction studies in which the $\gamma$-decay occurs between bound states, the $\gamma$-lifetime is very long compared to the direct-reaction time. Thus, the $\gamma$-decay occurs long after the interacting nuclei have separated, and the nuclear reaction affects the $\gamma$-decay only through the way it populates the different $m$-states of the $\gamma$-decaying level. However, the situation is quite different when the intermediate state is the GQR. Since the GQR is unstable with respect to compound-nucleus formation and to neutron emission, any $\gamma$-rays de-exciting the GQR must have been emitted within a time of the order of $10^{-22}$ sec. This is comparable to the time over which the

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$^c$ Fellow of the Comisión de Investigaciones Científicas, La Plata, Argentina.
direct reaction occurs. Thus, an appreciable fraction of the γ-rays are emitted while the $^{208}\text{Pb}$ and $^{170}\text{O}$ are still interacting with each other.

In sect. 2 we discuss the formalism for the reaction mechanism and we arrive at an expression for the angular distribution of the γ-rays decaying from the GQR. This expression contains incoherent sums over the excitation energies in the range of the GQR width and over the spin projection of the GQR states.

The calculations concerning the reaction process and their characteristic behaviour are reported in sect. 3. We also obtain the values of the substate occupation factors, which determine the magnitude and shape of the angular distribution of the emitted radiation.

In sect. 4 we present the final results using an approximate expression for the γ-ray angular distributions based on the smooth dependence of the occupation factors on the excitation energy in the range of the GQR width.

Finally, sect. 5 is devoted to conclusions.

2. The formalism

We use the distorted-wave Born approximation to generate the wave function that describes the relative motion of the separating $^{170}\text{O}$ and $^{208}\text{Pb}$ (GQR). We represent the loss of flux due to compound-nucleus formation and to neutron emission by adding a constant negative term, $-\Gamma$, to the imaginary part of the $^{170}\text{O}-^{208}\text{Pb}$ (GQR) optical potential. This simulates a decay probability per unit time of $\Gamma/\hbar$. We take $\Gamma$ to equal the observed GQR width of 2.4 MeV. We treat the $^{170}\text{O}$ projectile as if it were spherically symmetric and without possibility of excitation.

If $|\psi\rangle$ symbolizes the total state of the $^{170}\text{O}_{g.s.}-^{208}\text{Pb}$ system, the relative motion in the elastic and inelastic channels is given by

$$X_0(r) = \langle ^{208}\text{Pb}_{g.s.} \times ^{170}\text{O} | \psi \rangle = \frac{1}{r} \sum S_i(r) Y_0^i(\hat{r}), \quad (2.1a)$$

$$\psi_m(r) = \langle ^{208}\text{Pb}_{e,2,-m} \times ^{170}\text{O} | \psi \rangle = \frac{1}{r} \sum S_i^e(r) Y_m^i(\hat{r}). \quad (2.1b)$$

The integrations implied in the matrix elements on the left-hand side of eqs. (2.1) are performed over the internal coordinates of the $^{170}\text{O}$ and $^{208}\text{Pb}$ nuclei, for a constant relative value of the vector $r$. The labels $e, 2, -m$ refer, respectively, to the excitation energy, angular momentum and angular momentum projection of the GQR in $^{208}\text{Pb}$.

We make the usual assumption about the coupling between the nuclear deformation operator $\hat{\beta}_{2m}$ and the relative motion, based on the expansion of the real part of the deformed optical potential $U(r, R)$. The radius $R$ of such a Saxon-Wood-shaped potential is $R = R_0 + R_t \sum_m \beta_{2m} Y_m^2(\hat{r})$, where $R_0 = \frac{r_0(A_t^{1/3} + A_p^{1/3})}$ and $R_t = r_0 A_t^{1/3}$; $r_0$ is the radial optical model parameter (see table 1), and $A_t$ and $A_p$ are the masses of the target and projectile, respectively.
The expression for the expansion of the deformed potential around $R_0$ is

$$U(r, R) = U(r) - \left(\frac{dU}{dr}\right)_{R=R_0} R \sum_m \beta_m Y_m^2(r). \quad (2.2)$$

In eq. (2.2), $U(r) = U(r, R = R_0)$ is the central real optical potential, and the second term of the r.h.s. is the coupling potential $V$ between the elastic and inelastic channels (see appendix A.1).

If eqs. (2.1) and (2.2) are used in the Schrödinger equation, and if we ignore the effects of the inelastic channels on the motion in the elastic channel (apart from their contribution to the optical potential), we obtain the following equations for the radial functions (see appendix A.1):

\[
\begin{align*}
\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + k_0^2 - (2\mu/\hbar^2) \left[ U(r) + iW(r) \right] \right\} f_l(r) &= 0, \quad (2.3a) \\
\left\{ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + k_e^2 - (2\mu/\hbar^2) \left[ U(r) + iW(r) - \frac{1}{2}i\Gamma \right] \right\} g_{lm}^e(r) &= \mathcal{G}_{lm}^e(r). \quad (2.3b)
\end{align*}
\]

Here $k_0$ and $k_e$ are, respectively, the wave numbers in the elastic and inelastic channels. As discussed above, the constant term $-\frac{1}{2}i\Gamma$ in the excited-state optical potential simulates the loss of flux due to compound-nucleus formation and to neutron emission (see appendix A.2). The coupling function in eq. (2.3b) is given by

$$\mathcal{G}_{lm}^e(r) = \left( - \right)^{m+1} (2\mu/\hbar^2) R \left( \frac{dU}{dr} \right)_{R=R_0} \beta_e$$

$$\times \sum_{l'} \frac{1}{l'} f_{l'}(r) \langle 1020 | l'0 \rangle \langle l - m2m | l'0 \rangle,$$

$$\beta_e = \langle ^{208}\text{Pb}_{e,2} || \beta_2 || ^{208}\text{Pb}_{g.s.} \rangle \quad (2.4)$$

where

is the reduced matrix element. We require the solution of eq. (2.3a) whose incoming part at large $r$ is the same as the incoming part of a Coulomb-distorted plane wave (see eq. A.17)). The solution of eq. (2.3b) at large $r$ must consist of a decaying outgoing wave. This can be obtained in the usual way by constructing a Green function using the appropriate solutions of the homogeneous version of eq. (2.3b) (see appendix A.3).

Next we consider the $\gamma$-decay of the GQR as the $^{208}\text{Pb}$ and $^{170}\text{O}$ nuclei move past each other. The question now is the extent to which the motion of the $^{208}\text{Pb}$ nucleus affects the counting rate of photons in the $^{208}\text{Pb} - ^{170}\text{O}$ rest frame. We argue as follows that this effect is negligible. Since the recoiling $^{208}\text{Pb}$ nucleus is not detected, the total probability of detecting a photon with momentum $p_\gamma$ is given by an incoherent integral over all values of the recoil momentum $P_f$ of the $^{208}\text{Pb}$ after the emission of the photon, and hence over all initial momenta $P = p_\gamma + P_f$ of the excited $^{208}\text{Pb}$ nuclei, as measured in the $^{208}\text{Pb} - ^{170}\text{O}$ rest frame. The $^{208}\text{Pb} - ^{170}\text{O}$ relative wave function
in momentum space is dominated by $P$-values in the range
\[
(2\mu E_r)^{1/2} \lesssim P \lesssim (2\mu (E_r + U_0))^{1/2} = (2\mu E_r)^{1/2} + (\mu/(2E_r))^{1/2}U_0. \tag{2.6}
\]
Here $\mu$ is the $^{208}\text{Pb}-^{170}\text{O}$ reduced mass, $E_r$ is the asymptotic kinetic energy in the center-of-mass system, and $U_0$ is the depth of the real part of the optical potential.

The photon energy $E_\gamma$, as measured in the $^{208}\text{Pb}-^{170}\text{O}$ rest frame, is related to the photon energy $E'_\gamma$ in the rest frame of the $^{208}\text{Pb}$ nucleus, by a Doppler shift factor of the order of
\[
1 \pm v/c = 1 \pm P/M(208\text{Pb})c.
\]
Since $P$ varies over the range (2.6), the range of $E'_\gamma$ for a given $E_\gamma$ will be about
\[
\frac{\Delta P}{M(208\text{Pb})c} \approx E_\gamma(\mu/(2E_r))^{1/2}U_0/(M(208\text{Pb})c).
\]
This $E'_\gamma$ range is small compared to the width of the level,
\[
\frac{\Delta E'_\gamma}{\Gamma} \approx (E_\gamma/\Gamma)(\mu/(2E_r))^{1/2}U_0/(M(208\text{Pb})c)
\]
\[
\approx \frac{10.5 \text{ MeV}}{2.4 \text{ MeV}} \left(\frac{17/(2 \times 208 \times 225 \times 391)}{(2E_r)/2} \right)^{1/2}U_0/E_r^{1/2} \sim 0.005 \tag{2.7}
\]
for $E_\gamma \sim 351$ MeV and $U_0 \sim 50$ MeV. Since $\Gamma$ determines the range over which the $\gamma$-emission probability varies appreciably, the smallness of the ratio (2.7) implies that the probability for $\gamma$-emission, $T(P)$, is very nearly constant over the $E'_\gamma$ range covered when we do the incoherent integral over $P$ around an average value $P_0$ of its modulus. In this way the following factorization is possible:

\[
\int T(P)|\langle P|\psi_m\rangle|^2 d^3P = T(P_0) \int |\langle P|\psi_m\rangle|^2 d^3P. \tag{2.8}
\]

A further step is to replace the integral over $P$ by an incoherent integral over the relative vector $r$:

\[
\int |\langle P|\psi_m\rangle|^2 d^3P = \int |\langle \psi_m|r\rangle \langle r|P\rangle \langle P|r'\rangle \langle r'|\psi_m\rangle|^2 d^3P d^3r d^3r'.
\]

\[
= \int \delta(r-r')|\langle r|\psi_m\rangle|^2 d^3r d^3r'.
\]

\[
- \int |\langle r|\psi_m\rangle|^2 d^3r. \tag{2.9}
\]

The emitting nucleus is partially polarized with the occupation factors of each $m$-value given by eq. (2.9). We will rewrite them as $P^*_m(\beta)\beta^2$, where

\[
P^*_m = \int|^{208}\text{Pb}_{e,2} - m \times ^{17}\text{O}|\psi\rangle|^2 d^3r/(\beta^2)\beta^2
\]
\[
= \sum_I \int_0^\infty |g^m_{im}(r)|^2 dr/(\beta^2)\beta^2. \tag{2.10}
\]
The radial integrals in eq. (2.10) converge, due to the exponential decay of the functions \( |g_{I,m}(r)|^2 \) for large \( r \). We have explicitly separated in eq. (2.10) the dependence of the occupation parameters on the reduced matrix elements \( \beta^e \), which concerns the nuclear structure aspects of the problem. The net result is that the probability per unit time of detecting a photon with momentum \( p_\gamma \) in the \(^{17}\text{O}-^{208}\text{Pb} \) rest frame is the same as if that photon were emitted by excited \(^{208}\text{Pb} \) nuclei, populated according to the \( P_m^e \) factors, in their proper frame.

Now, it is possible to use the standard expressions for the electromagnetic decay of polarized nuclei. Thus the rate for the electromagnetic \( 2^\lambda \)-pole emission in the direction \( \theta \) is given by

\[
T^e_\lambda(\beta^e)^2 \sum_m P_m^e W_{\lambda m}(\theta). \tag{2.11}
\]

Here \( T^e_\lambda \) is the electromagnetic transition probability for radiation of multipolarity \( \lambda \) and frequency \( w^e = (\varepsilon - E_i)/\hbar \) emitted in a transition from an initial \( J_i = 2 \) state with energy \( \varepsilon \) to a final state with spin \( J_f \) and energy \( E_f \):

\[
T^e_\lambda = \sum_{\mu} T^{\lambda-\mu}_{J_i \rightarrow J_f} = \frac{8\pi(\lambda + 1)}{5\lambda[(2\lambda + 1)!!]} \frac{1}{\hbar} \left( \frac{w^e}{c} \right)^{2\lambda + 1} |\langle E_f J_f | \mathcal{M}_\lambda | \varepsilon, J_i = 2 \rangle|^2. \tag{2.12}
\]

\( \langle E_f J_f | \mathcal{M}_\lambda | \varepsilon, J_i = 2 \rangle \) is the reduced matrix element for the multipole electromagnetic operator \( \mathcal{M}_\lambda \), and

\[
W_{\lambda m}(\theta) = \frac{5}{4\pi} (-)^{J_i - m} (2\lambda + 1) \sum_{K \text{ even}} \langle \lambda 1\lambda - 1 | K 0 \rangle \langle 2m 2 - m | K 0 \rangle \times \begin{pmatrix} \lambda & 2 \\ \lambda & J_f \end{pmatrix} P_K(\cos \theta) \tag{2.13}
\]

is the normalized directional distribution of the radiation (\( \int W_{\lambda m}(\theta) d\Omega = 1 \)).

The total rate of decay per unit of solid angle in the direction \( \theta \) as induced by an incident beam of flux \( (2E_\gamma/\mu)^{1/2} \text{ [fm}^{-2}\text{.sec}^{-1}] \) is given by an incoherent summation of terms (2.11) over the intermediate excitation energies \( \varepsilon \) in the range of the giant resonance width

\[
S^e_\lambda(\theta) = \sum_\varepsilon T^e_\lambda(\beta^e)^2 \sum_m P_m^e W_{\lambda m}(\theta). \tag{2.14}
\]

If this width is sufficiently small compared to the range of variation of the factors \( P_m^e \), we can replace these quantities by an average value \( P_m \). If, in addition, we also use an average value \( \tilde{w} \) for the photon frequency, the total rate of decay can be split into three terms that correspond to the reaction mechanism, electromagnetic

* This separation is easily performed, since the source terms \( g_{I,m}^e(r) \) (eq. (2.4)) are simply proportional to \( \beta^e \) and thus the ratios \( g_{I,m}^e(r)/\beta^e \) are obtained by giving unit value to \( \beta^e \) in the solution of eqs. (2.3).
field and nuclear structure,

\[ S_\lambda(\theta) = \frac{8\pi(\lambda + 1)}{5\lambda[(2\lambda + 1)!!]^2} \frac{1}{\hbar} \left( \frac{w}{c} \right)^{2\lambda + 1} \sum_m P_m W_\lambda^m(\theta) \times \sum_{\varepsilon} (\beta^\varepsilon)^2 |\langle E_\varepsilon, J_\varepsilon | \mathcal{M}_\lambda | e, J_1 = 2 \rangle|^2. \]  

(2.15)

The matrix element in the last line of the expression (2.15) may be calculated using the method of ref. 4).

3. Calculation of the factors \( P_m \)

The following discussion is based on calculations which use the optical parameters suggested in ref. 5) (set 1 in table 1). The excitation energy in the inelastic channel is \( E_0 = 10.5 \, \text{MeV} \), corresponding to the centroid of the GQR. Appendix A describes the details of the calculation, and the special features associated with the presence of a constant absorptive term extending out to infinite \( r \).

In fig. 1 we show the \( l \)-dependence of the individual terms

\[ P_{lm} \equiv \int_0^\infty |g_{lm}(r)|^2 \, dr/(\beta^r)^2 \]  

(3.1)

of the sum (2.10). For \( m = 0 \) and \( m = 2 \), \( P_{lm} \) is rather strongly peaked at \( l = 155 \). The grazing partial wave is

\[ l_g = \left[ 2\mu \left( E - V_C(R_0) \right) \right]^{1/2} R_0 \approx 135 \]

where \( V_C \) is the Rutherford potential.

As expected, the inelastic reaction is dominated by \( l \)-values somewhat greater than \( l_g \), since absorption is greater for \( l < l_g \) than for \( l > l_g \). There is however, a finite contribution from \( P_{l,0} \) with \( l \)-values appreciably smaller than \( l_g \). Since these small \( l \)-values correspond to mostly incoming waves, they would make very little contribu-

### Table 1

<table>
<thead>
<tr>
<th>Optical-model parameters used in the calculations</th>
<th>Set 1</th>
<th>Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_0 ) [MeV]</td>
<td>-50.00</td>
<td>-20.00</td>
</tr>
<tr>
<td>( r_{oo} ) [fm]</td>
<td>1.10</td>
<td>1.34</td>
</tr>
<tr>
<td>( a_{oo} ) [fm]</td>
<td>0.80</td>
<td>0.57</td>
</tr>
<tr>
<td>( W_0 ) [MeV]</td>
<td>-50.00</td>
<td>-34.</td>
</tr>
<tr>
<td>( r_{oo} ) [fm]</td>
<td>1.10</td>
<td>1.34</td>
</tr>
<tr>
<td>( a_{oo} ) [fm]</td>
<td>0.80</td>
<td>0.57</td>
</tr>
<tr>
<td>( r_{oo} ) [fm]</td>
<td>1.864</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Standard Saxon-Wood potentials are used with nuclear radius \( R_x = r_o(A^{1/3} + A_p^{1/3}) \). The Coulomb potential corresponds to a uniformly-charged sphere of radius \( R_C = r_C A^{1/3} \).
Fig. 1. The partial occupation factors $P_{lm}$ for each projection $m$ of the angular momentum of the excited $^{208}\text{Pb}$ nuclei. The hatched zone indicates, in each case, the post-contact region contributions as discussed in sect. 3. A tentative boundary between the contact and post-contact regions is taken at 13 fm. The $P_{m}$ factors for set 1 are $P_{0} = 244.8$, $P_{1} = 193.1$ and $P_{2} = 343.4$.

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Fig. 2 and 3 show some typical source terms used in the inhomogeneous equation (2.3b), which determines the inelastic radial functions $g_{lm}(r)$. The fact that the source term for $l = 50$ is more oscillatory than for $l = 155$ is associated with the fact that the elastic wave function $f_{50}(r)$ in (2.4) has lower radial momentum than $f_{155}(r)$ does.

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Fig. 2 shows that the $l = 155$ source terms are concentrated in the radial range $10.5 \leq r \leq 13.0$ fm. Fig. 4 shows the radial probability densities for the solutions $g_{155,m}(r)$ associated with these source terms. These curves can be interpreted in terms of the excitation of the GQR when the colliding nuclei are in the range of the coupling interaction. As the nuclei separate and the GQR decays via compound-
Fig. 2. The source terms $g_{lm}(r)$ for $l=155$ and each projection $m$. Solid lines and dashed lines correspond to real and imaginary parts, respectively.

Fig. 3. The real part of the source terms $g_{lm}(r)$ for $l=50$ and each projection $m$. The corresponding imaginary parts only differ from the real parts in a negative shift of the oscillatory pattern which corresponds to a quarter wavelength. This corresponds to the purely incoming character of $f_{50}(r)$ in the contact region.
nucleus formation and neutron emission, \( |g_{155,m}(r)|^2 \) decreases with a characteristic decay length of approximately 31 fm. We choose \( r = 13 \) fm as the boundary between the "contact" and "post-contact" regions. At this separation the coupling potential has about 1% of its maximum value. The shaded regions of fig. 1 indicate the post-contact contribution to the probability, as a function of \( l \).

Figs. 3 and 5 show the comparable situation for \( l = 50 \). In this case the source terms are concentrated in the radial range \( 7.5 \leq r \leq 11.5 \) fm. However, the greater absorption for this region means that there is very little outgoing inelastic flux in the surface region. Thus fig. 5 shows that the existence of the GQR is confined to the contact regions where the excitation takes place. The GQR does not get out to the post-contact region, so the \( r > 13 \) fm tail present for \( l = 155 \) (fig. 4) is absent for \( l = 50 \). Fig. 1 also shows that post-contact existence of the GQR is confined to the higher \( l \)-values.

In order to evaluate the energy dependence of the \( P_m \) factors in the range of the giant-resonance-width calculations for excitation energies \( \epsilon_1 = 9.5 \) MeV and \( \epsilon_2 = 11.5 \)
MeV were performed. The ratios between the factors for energy $\epsilon_1$, with respect to the average energy $\epsilon_0$, are

$$P_{0}^{\epsilon_{1}}/P_{0}^{\epsilon_{0}} = 1.078, \quad P_{1}^{\epsilon_{1}}/P_{1}^{\epsilon_{0}} = 1.015, \quad P_{2}^{\epsilon_{1}}/P_{2}^{\epsilon_{0}} = 1.088. \quad (3.2)$$

Similar values are obtained for the ratios of the factors $P_{m}^{\epsilon_{0}}/P_{m}^{\epsilon_{2}}$. Analysis of the influence of the excitation energy on the partial contributions $P_{lm}$ shows that such a dependence is only relevant for orbital angular momenta corresponding to the peak around $l \sim 155$.

Calculations were also performed using the parameters of set 2 in table 1 in order to evaluate the dependence of the polarization effects on the optical-model parameters. This set is just a reasonable heuristic choice. In particular, the value for the diffuseness was taken closer to the usual ones. The corresponding value in set 1 seems to be rather large, leading to an unusual extended nuclear surface.

The polarizations $p_{i} = P_{i}/\Sigma P_{i}$ for set 2 are

$$p_{0} = 0.314, \quad p_{1} = 0.224, \quad p_{2} = 0.462. \quad (3.3)$$

They are not too different from those obtained for set 1, which are shown in the second row of table 2.

Finally, we analyze the dependence of polarization effects on the value of the decay width. Calculations for $\Gamma_1 = 0.48$ MeV and $\Gamma_2 = 24$ MeV, corresponding to decay lengths $|g_{im}|^2 \sim e^{-r/\lambda^0}$, $\lambda_1^D = 156$ fm and $\lambda_2^D = 3.1$ fm, were performed. The decay length that corresponds to the giant quadrupole resonance in $^{208}$Pb is $\lambda^D = 31.2$ fm.

Table 2 shows the total (contact plus post-contact) and post-contact contributions to the polarizations $p_m$ for the different $\Gamma$-values. The ratios between the post-con-
TABLE 2
Polarizations for different resonance widths

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Gamma [\text{MeV}] = 2.4 )</td>
<td>0.48</td>
<td>24.0</td>
</tr>
<tr>
<td>( P_m^T )</td>
<td>0.313</td>
<td>0.314</td>
<td>0.336</td>
</tr>
<tr>
<td>( P_{PC}^T )</td>
<td>0.325</td>
<td>0.319</td>
<td>0.335</td>
</tr>
<tr>
<td>( P_m^T / P_m^{PC} )</td>
<td>0.423</td>
<td>0.767</td>
<td>0.024</td>
</tr>
</tbody>
</table>

PC = post contact; T = total.

Contact and the total factors \( P_m \) are also shown in Table 2. These results seem to indicate that the resonance width \( \Gamma = 2.4 \text{ MeV} \) of the GQR in \( ^{208}\text{Pb} \) is quite adequate for studying possible effects on the \( \gamma \)-ray emission due to the proximity of the decaying nucleus and the projectile. In this case, the contact and post-contact regions contribute in similar proportions.

4. Angular distributions of the \( \gamma \)-rays

In sect. 3 we noted the weak excitation energy dependence of the \( P_m \) in the energy range of the GQR. It is thus a reasonable approximation to ignore this energy dependence, in which case the expression (2.14) for the photon flux can be written as a product of two factors:

\[
\delta_\lambda = \sum_\varepsilon (\beta^\varepsilon)^2 T_\lambda^\varepsilon, \quad (4.1a)
\]

\[
\gamma_{\lambda, J_f}(\theta) = \sum_m P_m^{\nu_\lambda} W_{\lambda m}(\theta). \quad (4.1b)
\]

The factor \( \delta_\lambda \) depends on the coupling between the GQR and the electromagnetic field, summed over the entire GQR width. This factor is independent of the polarization of the GQR. The factor \( \gamma_{\lambda, J_f}(\theta) \) is strongly dependent on polarization and determines the angular distribution. We will refer to it as the "reduced angular distribution".

Fig. 6 shows reduced angular distributions for the photon decay to the \( 0^+, 1^- \) and \( 3^- \) final states of \( ^{208}\text{Pb} \). Since \( \gamma_{\lambda, J_f}(\theta) \) is independent on the photon frequency \( \omega^\varepsilon \), these reduced distributions correspond to any \( 0^+, 1^- \) or \( 3^- \) final states. The reduced angular distributions are symmetric respect to \( \theta - \frac{\nu}{2} \) due to the presence of only even Legendre polynomials in eq. (2.14).

The solid lines of fig. 6 were calculated using the \( P_m \) factors evaluated for the optical parameters of set 1 in Table 1, while the dashed lines correspond to the parameters of set 2. Since the strengths of the potentials of set 2 are quite arbitrary, the two types of curves are normalized at \( \theta = 0^\circ \). The \( 0^+ \) state shows some changes
(about 12% in the maximum at 60°) due to the use of different optical parameters sets. For the decay to the 1 and 3 states, the influence of the chosen optical parameters is still less important. This behaviour can be explained with the help of fig. 7 which shows the angular distribution of each m-component of $\gamma_{\lambda, J}(\theta)$ for the 0+, 1− and 3− states. For the case of the decay to the 0+ state, each m-component has a very different angular shape. Then, in the final m-sum, each of them plays an important role for all angles $\theta$. As a consequence small changes in the ratios of the $P_m$ factors may produce changes in the angular distribution for this final state. The same fig. 7 shows the angular distribution of the m-components for the decay to the 1− and 3− states. In both cases, the $m = 2$ component has a net predominance over the $m = 1$ and $m = 0$ components in essentially the full angular range. Then, the total sum over $m$ is generally dominated by the $m = 2$ component and small changes in the ratios of the $P_m$ values produce small changes in the reduced angular distributions.
The differences between the $0^+$, $1^-$ and $3^-$ angular distributions could give us useful information about the reaction process. While the distributions for the $1^-$ and $3^-$ states are mainly isotropic, those corresponding to the $0^+$ increase from $0^\circ$ to $90^\circ$ with a smooth maximum around $\theta = 60^\circ$. These observations can be expressed in a simple form by means of the anisotropy ratios $\mathcal{A}_{J_f} = \gamma_{J_f} \gamma_{J_f}(\theta = 90^\circ) / \gamma_{J_f} \gamma_{J_f}(\theta = 0^\circ)$. We calculate

$$\mathcal{A}^{0^+} = 1.40, \quad \mathcal{A}^{1^-} = 0.87, \quad \mathcal{A}^{3^-} = 0.96 \quad (4.2)$$

using the parameters of set 1, and

$$\mathcal{A}^{0^+} = 1.54, \quad \mathcal{A}^{1^-} = 0.84, \quad \mathcal{A}^{3^-} = 0.95 \quad (4.3)$$

for the parameters of set 2. From the comparison between the ratios (4.2) and (4.3), we notice that the use of two very different sets of optical parameters leads, in the case we are analyzing, to quite similar angular distribution shapes.
Fig. 6 also shows the reduced angular distributions using the $P_m$ values calculated with the parameters of set 1 and taking into account only the contributions of the post-contact region (see fig. 1). Comparison of the ratios

$$
\mathcal{A}_\text{post-contact}^0 = 2.83, \quad \mathcal{A}_\text{post-contact}^1 = 0.74, \quad \mathcal{A}_\text{post-contact}^3 = 0.92
$$

(4.4)

for the post-contact contribution with those given by the relations (4.2) for the total region shows that drastic changes occur in the anisotropy of the angular distribution for the decay to the $0^+$ state. The post-contact zone also emphasizes the anisotropy of the $1^-$ state distribution but leaves practically unchanged the $3^-$ distribution. The post-contact angular distributions are dominated by the $m = 2$ contribution. The fact that the photon angular distribution is sensitive to the $^{17}$O-$^{208}$Pb separation at the time of photon emission opens up the interesting possibility of using the observed angular distributions to elucidate a more detailed picture of the geometry of the reaction. It would be useful, in this connection, to have angular distribution data for the de-excitation of the GQR produced via inelastic excitation of other projectiles and at several incident excitation energies.

5. Conclusions

In this paper we have proposed a formalism that allows us to calculate the angular distributions of $\gamma$-rays produced in the decay of the GQR after its excitation by inelastic scattering.

The analysis of the special reaction mechanism involved in the description of the excitation of the GQR and the subsequent electromagnetic decay from a source in-flight, while the projectile and the residual nucleus are close together, involves the study of several interesting effects. The formalism permits us to compare nuclear structure calculations, such as those presented in ref. 4), with experimental data. On the other hand, the analysis of the shapes of the angular distributions may give spectroscopic information. Changes in angular distributions, due to the fact that the emission comes either from the contact or post-contact region, indicate a possible approach in the study of nuclear structure of systems where two nuclei are under mutual interaction.

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Appendix A

THE EQUATIONS FOR THE RELATIVE MOTION

A.1. The coupled differential equations. If a plane wave travelling along the $z$-axis is scattered by a $J = 0$ target, which can be excited to a $J = 2$ state as a result of an
inelastic collision, the system may be described by the wave function
\[ \psi = \varphi_{g.s.} \chi_0 (r) + \sum_m \varphi_{-m} \psi_m (r). \]  
(A.1)

The wave function \( \psi \) is a solution of the Schrödinger equation
\[ (H_0 + V)\psi = E\psi, \]  
(A.2)

where
\[ H_0 = H_{\text{int}} + H_{\text{rel}} \]  
(A.3)
is an unperturbed hamiltonian, \( V \) is the perturbation that leads to the excited states, and \( E \) is the total energy of the system.

The intrinsic \( \varphi_i \) wave functions are solutions of the equation
\[ H_{\text{int}} \varphi_i = E_i \varphi_i. \]  
(A.4)

The hamiltonian for the unperturbed relative motion is given by
\[ H_{\text{rel}} = -\left( \frac{\hbar^2}{2\mu} \right) \nabla^2 + \left[ \frac{U(r) + iW(r)}{2} - \frac{i}{2} \delta \Gamma \right] \]  
(A.5)

where \( \delta = 0 \) for \( \chi_0 \) and \( \delta = 1 \) for \( \psi_m \). The elastic and inelastic channels are assumed to have the same optical potential, except for a constant negative imaginary term in the inelastic channel which simulates the loss of GQR flux due to compound-nucleus formation and neutron decay.

Projecting eq. (A.2) on the \( \varphi_{g.s.} \) and the \( \varphi_{-m} \) intrinsic states and neglecting transitions among the excited states we obtain the equations
\[ \left\{ \begin{array}{ll} E + \left( \frac{\hbar^2}{2\mu} \right) \nabla^2 - \left[ U(r) + iW(r) \right] \right\} \chi_0 (r) = 0, \\ E - E_i + \left( \frac{\hbar^2}{2\mu} - \left[ U(r) + iW(r) - \frac{i}{2} \delta \Gamma \right] \right) \psi_m (r) = \langle \varphi_{-m} | V | \varphi_{g.s.} \chi_0 (r) \rangle. \]  
(A.6)

We choose for the coupling potential \( V \) the first order of an expansion of the optical potential in powers of the deformation operator \( \beta_{2\mu} \) (the second term in the r.h.s. of eq. (2.2)). Therefore, the r.h.s. of (A.7) becomes
\[ \langle \varphi_{-m} | V | \varphi_{g.s.} \chi_0 (r) \rangle = -R_s (\partial U(r)/\partial r)_{R_{-R_0}} \sum_{\mu} \langle \varphi_{-m} | \beta_{2\mu} | \varphi_{g.s.} \rangle Y_{2\mu}^2(\hat{r}) \chi_0 (r) \]  
\[ = -R_s (\partial U(r)/\partial r)_{R_{-R_0}} \frac{1}{5} \beta^* Y_m^2(\hat{r}) \chi_0 (r), \]  
(A.8)

where \( \beta^* \) is the reduced matrix element (2.5).

We introduce the partial-wave expansion of the wave function for the relative motion
\[ \chi_0 (r) = \frac{1}{r} \sum_l f_l (r) Y^l_m (\hat{r}), \quad \psi_m (r) = \frac{1}{r} \sum_l g_l^m (r) Y^l_m (\hat{r}). \]  
(A.9)

In all cases \( Y_m^l = i^l Y^l_m \), where the \( Y^l_m \) are the usual spherical harmonics. With the aid
of the expression for the integral of the product of three spherical harmonics\(^6\)) and eqs. (A.7), (A.8) and (A.9), we obtain eqs. (2.3) and (2.4).

**A.2. The imaginary decay potential.** The magnitude of the constant imaginary term in eq. (A.5) can be evaluated as follows. Let the time-dependent Schrödinger equation including a constant imaginary term \(iC\psi\) be

\[
i\hbar \frac{\partial \psi}{\partial t} = -\left( \hbar^2/2\mu \right) \nabla^2 \psi - iC\psi.
\]

(A.10)

Multiplying to the left by \(\psi^*\), taking the complex-conjugate equation and subtracting, we obtain

\[
i\hbar \left( \psi^* \frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi^*}{\partial t} \right) = -\left( \hbar^2/2\mu \right) (\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*) - i2C\psi^*\psi
\]

\[
= -\left( \hbar^2/2\mu \right) \text{div}(\psi^* \nabla \psi - \psi \nabla \psi^*) - i2C\psi^*\psi.
\]

(A.11)

This result can be expressed as a continuity equation in terms of a density \(\rho\), a current \(j\) and a decay rate \(\lambda\):

\[
\frac{\partial \rho}{\partial t} + \text{div}j = -2C\rho/\hbar = -\lambda\rho.
\]

(A.12)

We now make the following identifications:

\[
\lambda = \Gamma/\hbar = 2C/\hbar,
\]

(A.13)

\(\Gamma\) being the corresponding resonance width. Therefore,

\[
C = \frac{1}{2} \Gamma,
\]

(A.14)

as used in the imaginary decay potential of eq. (A.5).

**A.3. The solutions of the equations.** The \(g_{lm}(r)\) partial-wave functions are obtained by means of the Green function method:

\[
g_{lm}(r) = b_l(r) \int_0^r a_l(r') \mathcal{G}_{lm}(r') \, dr' + a_l(r) \int_r^\infty b_l(r') \mathcal{G}_{lm}(r') \, dr'.
\]

(A.15)

The independent homogeneous-equation solutions of eq. (2.3b), \(a_l(r)\) and \(b_l(r)\), behave as \(a_l(r) \to 0\) \((r \to 0)\) and \(b_l(r) \to\) purely outgoing wave \((r \to \infty)\) and are normalized so that the wronskian is equal to unity:

\[
a_l(r) b'_l(r) - b_l(r) a'_l(r) = 1.
\]

(A.16)

The \(f_l(r)\) partial waves are properly normalized to describe asymptotically an incoming unitary Coulomb-distorted plane wave travelling along the z-axis plus an outgoing spherical wave:

\[
f_l(r) \to \left(\sqrt{4\pi}/k\right) \hat{t} e^{i\alpha} \left\{ F_l(r) - \frac{e^{2i\delta_l} - 1}{2i} (G_l(r) + iF_l(r)) \right\}.
\]

(A.17)
where \( F_i(r) \) and \( G_i(r) \) are the regular and irregular Coulomb wave functions, \( \sigma_i \) are the Coulomb phase shifts, and \( \delta_i \) the phase shifts including nuclear interactions as well as Coulomb effects different from those due to a point electric charge.

The imaginary potential extended throughout the whole space leads to some peculiarities in the behaviour of \( a_i(r) \) and \( b_i(r) \). In fact, the amplitude of the solution regular at the origin, \( a_i(r) \), increases without limit as the radius increases and, asymptotically, behaves as a spherical incoming wave. The outgoing component, which is present in the central region, vanishes as it travels outwards.

The calculation of the outgoing spherical wave, \( b_i(r) \), poses some practical problems. It is necessary to integrate inwards, starting from asymptotic values characteristic of purely outgoing waves, \( \frac{db_i(r)}{dr} = -b_i(r) \). An outward integration path in an extended imaginary negative potential would generate an explosive spurious incoming component. The origin of this component is the unavoidable lack of precision in the choice of starting values in the numerical calculation.

On the other hand, since in the present case the absorptive constant potential is moderate (i.e. the wavelength is rather small in comparison with the decay length), some care must be taken to avoid spurious ripple-like effects in \( b_i(r) \). With this purpose, we integrate the outgoing wave inwards through a conveniently intense negative imaginary potential located in the asymptotic region. This potential acts like a filter over a range of a few wavelengths.

In typical cases, the integration for the \( f_i(r) \) functions is carried out from the origin up to 40 fm, where the normalization of eq. (A.17) is done. For the optical parameters of set 1 and \( \Gamma = 2.4 \) MeV the integration for \( b_i(r) \) is made from 80 fm down to 5.5 fm, and that for \( a_i(r) \) is made from the origin up to 25 fm where the normalization to unitary wronskian is performed. The chosen integration step is 0.022 fm, corresponding to the description of a typical wave of 0.4 fm length by means of about 18 points. The decaying length of the wave functions corresponding to the neutron decay is about 31 fm.

In the integration for \( b_i(r) \), the imaginary potential filter is set with an intensity \(-50\) MeV between 73 and 77 fm with diffuseness of 0.5 fm. Sharp steps (i.e. zero diffuseness) introduce annoying oscillatory effects.

The \( g_{lm}(r) \) functions are calculated by means of eq. (A.20). The integrations are carried out using the complete equation in the region around the nuclear surface (from 5.5–8.5 fm to 14.8–14.5 fm, depending on the optical parameter set). From the upper limit of this surface region up to a distance (~80 fm) that includes a convenient number of the asymptotic decay lengths associated with the \( b_i(r) \) function, the approximation

\[
g_{lm}(r) = b_i(r) \left[ \int_{r_{\text{low}}}^{r_{\text{upp}}} a_i(r') g_{lm}(r') \, dr' \right] = \text{const} \times b_i(r) \quad (A.18)
\]

has been used. From this point on, we consider that the \( g_{lm}(r) \) behave as exactly-decaying exponential functions and we integrate analytically up to infinity.
References

2) N. Austern, in Direct nuclear reactions (New York, Wiley, 1970)