A heavy nucleus (like $^{208}\text{Pb}$) is 18 orders of magnitude smaller and 55 orders of magnitude lighter than a neutron star!

Yet bounded by a common entity, the nuclear Equation Of State (EOS)!

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The Robert A. Welch Foundation: Grant Number A-1266 and,
The Department of Energy: Grant Number DE-FG03-93ER40773
Symmetry Energy in the Equation of State of Asymmetric Nuclear Matter

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Softer EOS leads to a smaller star mass and radius for a given central density.
Observables sensitive to the asymmetry term in the EOS

Moderate density ($\rho < 1.5 \rho_o$):
- Fragment isotope distribution, isotopic & isobaric yield ratios
- Isospin distillation/fractionation, relative n & p densities
- Isospin diffusion
- Nuclear stopping & N/Z equilibration
- Pre-equilibrium emission
- Particle - particle correlation
- Light cluster production

High density ($\rho > 1.5 \rho_o$):
- Collective flow
- Subthreshold particle production
An inhomogeneous distribution of the neutrons and protons within the system is predicted, resulting in a dilute neutron rich (N/Z > 1) gas (light clusters) and a dense and symmetric (N/Z ~ 1) liquid (heavy fragments).

Studying isospin distillation

Measure the yields of the light clusters (gas phase)

Determine the n & p densities

Compare them from one reactions to another with different isospin (N/Z)

$^{58}\text{Ni} + ^{58}\text{Fe}$ (reaction 2, N/Z = 1.15)

&

$^{58}\text{Ni} + ^{58}\text{Ni}$ (reaction 1, N/Z = 1.07)

$^{58}\text{Fe} + ^{58}\text{Fe}$ (reaction 2, N/Z = 1.23)

&

$^{58}\text{Ni} + ^{58}\text{Ni}$ (reaction 1, N/Z = 1.07)

Relative n & p density @ 30, 40, 47 MeV/nucl.
If the clusters in reaction 2 are more neutron-rich than in reaction 1

Relative n density, \[
\left( \frac{\rho_{n,2}}{\rho_{n,1}} \right) > 1
\]

Relative p density, \[
\left( \frac{\rho_{p,2}}{\rho_{p,1}} \right) < 1
\]
Ratio of isotopic yields
$^{58}\text{Ni}, \; ^{58}\text{Fe} + ^{58}\text{Ni}, \; ^{58}\text{Fe}$ 30 MeV/nuc

$$R_{21}(N,Z) = C e^{\alpha N + \beta Z}$$
Relative neutron and proton densities at 30 MeV/nuc

\[ \frac{\rho_n}{\rho_N^{Ni+Ni}} \]

\[ \frac{\rho_p}{\rho_p^{Ni+Ni}} \]

Isotope ratios

Isotone ratios

N / Z
Relative neutron and proton densities at 30 MeV/nuc

\[ \frac{\rho_n}{\rho_{n}^{Ni+Ni}} \]

\[ \frac{\rho_p}{\rho_{p}^{Ni+Ni}} \]

Isotope ratios

Isotone ratios

N / Z
neutron and proton densities at 30, 40, 47 MeV/nuc

\[ \frac{\rho_n}{\rho_n^{Ni+Ni}} \]

\[ \frac{\rho_p}{\rho_p^{Ni+Ni}} \]
Experimental SMM model comparison

Isotope ratios

Isotone ratios

N / Z

Excitation energy (MeV/A)

Fe + Ni

Fe + Fe

ρₙ

ρₚ

ρₙ

ρₚ

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Temperature dependence of the scaling parameter $\alpha$

$^{58}\text{Fe} + ^{58}\text{Fe} / ^{58}\text{Ni} + ^{58}\text{Ni}$

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.372</td>
<td>-0.395</td>
</tr>
<tr>
<td>40</td>
<td>0.269</td>
<td>-0.372</td>
</tr>
<tr>
<td>47</td>
<td>0.23</td>
<td>-0.32</td>
</tr>
</tbody>
</table>

$\alpha T = 4C_{sym} \left( \frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2} \right)$

Tsang PRC64, 054615 (2002)
Symmetry energy and the fragment yield distribution in Multifragmentation reaction

\[ \alpha T = 4C_{sym}\left(\frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2}\right) \]

EOS and dynamical simulation of fragment production (AMD model calculations)

\[ V_{\text{Gogny-AS}} = V_{\text{Gogny}} - (1 - x) t_3 [\rho(r_1)^{1/3} - \rho_0^{1/3}] P_\sigma \delta(r_1 - r_2) \]

Symmetry energy and the scaling parameter $\alpha$

$C_{\text{sym}} \sim 18 - 20 \text{ MeV} \ ; \ \rho \sim 0.08 \text{ fm}^3$

$\alpha T = 4 C_{\text{sym}} \left( \frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2} \right)$

Symmetry energy and the scaling parameter $\alpha$

$C_{\text{sym}} \sim 18 - 20 \text{ MeV} ; \rho \sim 0.08 \text{ fm}^{-3}$

Formation of hot neutron rich nuclei in supernova explosion

During supernova II type explosion the thermodynamical conditions of stellar matter between the protoneutron star & the shock front correspond to nuclear liquid-gas coexistence region. Neutron rich hot nuclei can be produced in this region which can influence the dynamics of the explosion contribute to the synthesis of heavy elements.

A slight decrease in the symmetry energy co-efficient can shift the mass distribution to higher masses

Deep Inelastic Transfer mechanism can produce neutron-rich heavy residues.
FAUST has solid angle coverage of 90% from 2.31° to 33.63°, 71% from 1.6° to 2.3°, and 25% from 33.6° to 44.9°.

http://cyclotron.tamu.edu/sjygroup/external/Detectors/FAUSTWeb/faust.html
Production of quasi projectiles over a wide range in N/Z from 33MeV/nuc $^{20}\text{Na}$, $^{20}\text{Ne}$, $^{20}\text{F}$ + Au reactions

Isobar Isoscaling

$R_{21} = C \exp(\alpha'A + \beta'(N-Z))$

$$\alpha' = \Delta(\mu_n + \mu_p)/2T$$

$$\beta' = \Delta(\mu_n - \mu_p)/2T$$

$^{28}\text{Si} + {}^{112,124}\text{Sn}$

30 MeV/nuc

$\beta' = 0.237 \pm 0.038$

50 MeV/nuc

$\beta' = 0.186 \pm 0.017$
Botvina Isoscaling

\[ R_{21} = C \exp(\alpha' A + \beta'(N-Z)) \]

\[ ^{28}\text{Si} + ^{112,124}\text{Sn} ; 50 \text{ MeV/nuc} \]

<table>
<thead>
<tr>
<th>( E^* ) (MeV)</th>
<th>( \beta' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.31</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>0.18</td>
</tr>
<tr>
<td>7</td>
<td>0.17</td>
</tr>
</tbody>
</table>
\[ \beta' = \frac{\Delta(\mu_n - \mu_p)}{2T} \]
Al quasiprojectiles
MARS Recoil Separator and Setup

 Silicon Telescope:
   \( \Delta E_1, \ X, Y \) (Strips)
   \( \Delta E_2, \ E_{\text{residual}} \)

 MARS Acceptances:
   Angular: 9 msr
   Momentum: 4 %

 Beam angle set to:
   0° (1.0-3.0°) for Kr+Ni \( (\theta_{\text{gr}} = 3.5°) \)
   4° (2.5-5.5°) for Kr+Sn \( (\theta_{\text{gr}} = 6.5°) \)
The Superconducting Solenoid Rare Isotope Line at TAMU:

Schematic diagram of the setup for heavy-residue studies from DIC:

- Beam from K500 Cyclotron
- Target Chamber
- Vacuum Gate Valves
- Beam Blocking and Angular Acceptance Definition: 1.5-3.0 deg. 2.0-6.0 deg
- U. Mich. 7-Tesla Superconducting Solenoid Magnet “BigSol”
- Intermediate Image Diagnostics/Detector Box
- Al degrader
- Quadrupole Triplet
- Final Image Detector Box
- P/Q Aperture 24,12,6 mm holes T,X,Y
- P/Q Aperture 24,12,6 mm holes T,X,Y
- ORTEC Si Telescope ΔE1,ΔE2,E1,E2 (In/Out)
- TOF path (7.5 meters)
DIT mechanism can produce neutron-rich heavy residues

Data using targets:
- \(^{124}\text{Sn}\)
- \(^{112}\text{Sn}\)
- \(^{64}\text{Ni}\)

--- EPAX2

Mass Distribution of Germanium from \(^{86}\text{Kr}(25\text{MeV/u}) + 124\text{Sn}, 112\text{Sn}, 64\text{Ni}\)

Target | N/Z | Valley of stability
---|---|---
\(^{124}\text{Sn}\) | 1.48 | n-rich \(^{118}\text{Sn}\)
\(^{112}\text{Sn}\) | 1.24 | n-poor
\(^{64}\text{Ni}\) | 1.29 | n-rich \(^{60}\text{Ni}\)

Comparison: Data/EPAX : $^{86}$Kr (25 MeV/u) + $^{64}$Ni, $^{124}$Sn

For near-projectile fragments and above the projectile N/Z, the cross sections are larger.

DIC between massive n-rich nuclei appear to be advantageous for very high N/Z RIB production.
Scaling of Yield Ratios:

\[ R_{21}(N,Z) = \frac{Y_2}{Y_1} \]

- \(^{86}\text{Kr} + ^{124}\text{Sn}, ^{112}\text{Sn}\) (data inside \(\theta_{gr} = 6.2^\circ\))

- \(^{86}\text{Kr} + ^{64}\text{Ni}, ^{58}\text{Ni}\) (data outside \(\theta_{gr} = 3.5^\circ\))
Isoscaling Parameter $\alpha$ : *

$\alpha = 0.43$
$\alpha = 0.27$

$R_{21} = C \exp (\alpha N)$

$\alpha = 4 \frac{C_{\text{sym}}}{T} (\frac{Z}{A})_1^2 - (\frac{Z}{A})_2^2$

Quasi-projectiles 1: n-poor 2:n-rich

Isoscaling data from residues of $^{64}$Ni (25MeV/nucleon)

\[ R_{21} (N,Z) = \frac{Y_2}{Y_1} \]

BigSol data

\[ R_{21} = C \exp (\alpha N) \]
\[ \alpha = 4C_{\text{sym}}/T \left( (Z/A)_1^2 - (Z/A)_2^2 \right) \]

\begin{itemize}
  \item \textit{86Kr, 64Ni, 136Xe data: Isocaling parameter }\textit{\alpha vs }\Delta(Z/A)^2: \textit{N/Z equilibrated, no effect of pre-eq.}
  \item Quasi-projectiles: E/A \sim 20-25 MeV
  \item \textbf{86Kr} + 124, 112Sn \hspace{1cm} \varepsilon^* \cong 2.0 \text{ MeV/u}
  \item \textbf{86Kr} + 64, 58Ni \hspace{1cm} \varepsilon^* \cong 2.4 \text{ MeV/u}
  \item \textbf{64Ni} + 124, 112Sn \hspace{1cm} \varepsilon^* \cong 2.9 \text{ MeV/u}
  \item \textbf{64Ni} + 64, 58Ni \hspace{1cm} \varepsilon^* \cong 2.9 \text{ MeV/u}
  \item \textbf{64Ni} + 232Th, 208Pb \hspace{1cm} \varepsilon^* \cong 2.5 \text{ MeV/u}
\end{itemize}
Variation w.r.t excitation energy:

Data:
- $^{86}\text{Kr} + ^{124,112}\text{Sn}$
- $^{86}\text{Kr} + ^{64,58}\text{Ni}$
- $^{64}\text{Ni} + \text{Ni,Sn,Th-Pb}$
- $^{136}\text{Xe} + \text{Ni,Sn,Th-Au}$

Calculation:
- Fermi Gas (K=13)
- Mononucleus expansion model (L. Sobotka, J. Toke)

\[ C_{\text{sym}} = c \frac{T}{4} \]
Summary

- Projectile residue distributions from peripheral to mid-peripheral collisions show enhanced production of neutron–rich nuclei.
- Heavy-Residue Isoscaling can be used as probe of $E_{sym}$.
- Multifragmentation reactions may be able to distinguish between various functional forms of the density dependence of the symmetry energy.
- Symmetry energy can be significantly below saturation density and could be interesting in the studies relating to the elemental abundances in core collapse supernova explosion.

- Effect of Secondary Decay on isoscaling parameters needs to be understood.

- Comparisons with Reaction Models:
  - Extension of studies using higher $\Delta N/Z$ beams and heavy targets.