Isospin Transport at Fermi Energies

V. Baran, M. Colonna, M. Di Toro LNS Catania, Italy
M. Zielinska-Pfabe Smith College Northampton, MA, USA
H.H. Wolter University of Munich, Germany
Rationale

• Increased interest in nuclear reactions at Fermi energy caused by more detailed experimental data allowing a deeper investigations of reaction mechanisms.

• Study of the properties and effects of the asymmetry term of the EOS away from saturation.

• Central collisions-effect of distillation/fractionation.

• Semi-peripheral collisions-Isospin dynamics.
Goals
Isospin transfer through the neck--role of pre-equilibrium emission, isospin diffusion due to the N/Z ratio and isospin drift due to the density gradient; their dependence on asy-stiffness of EOS.

Methods
BUU simulations with inclusion of fluctuations for Sn + Sn collisions at 50MeV/u at b = 6,8,9,10 fm. Two kinds of density dependence of the asymmetry term in EOS:

\[ E_{\text{sym}}(\rho, I) \frac{I^2}{A} = C_{\text{sym}}(\rho)I^2 \quad I = \frac{N - Z}{A} \]

1) asy-superstiff \( \sim \rho^2 \)
2) asy-soft (SKM) saturating above normal density.

Results and Their Interpretation
Conclusions
Mean Trajectory - the formalism

Many body systems - the best we can do is mean field approach.

Introducing HF and one body potential \( U(\vec{r}) \) and single particle density matrix \( \rho(\vec{r}, \vec{r}') \) one has to solve the TDHF equation

\[
 i\hbar \dot{\rho} = [\hbar, \rho]
\]

\( \hbar \)- single body hamiltonian

TDHF is complicated and time consuming

If collisions are included, it implies that extended TDHF formalism with Boltzmann collision term.

Applying Wigner Transform to the one body density matrix \( \rho(\vec{r}, \vec{r}') \) we get the distribution function
\[ f(\vec{r}, \vec{p}) = \int d^3\vec{s} e^{-\frac{\vec{p} \cdot \vec{s}}{\hbar}} \rho(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2}) \]
\[
(\vec{s} = \vec{r} - \vec{r}')
\]

With the HF potential \( U = U(\vec{r}) \) the TDHF equation is transformed to:

\[
\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}_r f - \sum_{n=0}^{\infty} \left( \frac{\hbar}{2i} \right)^{2n} \frac{1}{(2n+1)!} \vec{\nabla}_r^{(2n+1)} U(\vec{r}) \vec{\nabla}_p^{(2n+1)} f = 0
\]

If one keeps only the first term in the \( \hbar \) expansion

\[
\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}_r f - \vec{\nabla}_r U \vec{\nabla}_p f = 0 \quad \text{VE}
\]

[If \( U(\vec{r}) \) is harmonic, VE is identical with the exact Wigner transformation of TDHF, validity of VE to describe quantal phenomena is related to the validity of the local harmonic approximation for U]

We start with 2 nuclei moving towards each other (in cm frame). Each nucleus is represented by many pseudo or test particles - gaussians
\[ f(\vec{r}, \vec{p}, t) = \sum_{i=1}^{N} f_i(\vec{r}_i, \vec{p}_i, t) \]

\[ f_i = \frac{1}{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r-r_i)^2}{2\sigma^2}} e^{-\frac{(p-p_i)^2}{2\sigma^2^2}} \]

The widths \( \sigma_r \) and \( \sigma_p \) are chosen to minimize

\[ \left( \sqrt{\langle r \rangle^2} - \sqrt{\langle r \rangle_{\text{exp}}^2} \right)^2 \]

nuclear radius

and

\[ \left( B - B_{\text{exp}} \right)^2 \]

binding energy

We have to solve the equation of motion

\[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}_r f - \vec{\nabla}_r U \vec{\nabla}_p f = I_{\text{coll}} \]

\( U \) - potential energy

\( I_{\text{coll}} \) describes collisions between test particles.
Skyrme Interaction

$$U(\rho) = A (\rho/\rho_0) + B (\rho/\rho_0)^\sigma + C \rho / (2\rho_0) (\rho_n - \rho_p) \tau$$

$$A = 356 \text{ MeV, } \quad B = 303 \text{ MeV, } \quad \sigma = 7/6$$

$$\tau = \begin{cases} +1 & \text{n} \\ -1 & \text{p} \end{cases}$$

( it is a soft EOS with compressibility of 200 MeV )

Two different choices were made for the density dependence of the symmetry term:

$$C = 32 \text{ MeV} \quad - \text{asy stiff}$$

$$C/\rho_0 = a + b \rho \quad \text{with } a = 481.7 \text{ MeV fm}^3$$

$$B = -1638.2 \text{ MeV fm}^6 \quad - \text{asy soft}$$
This equation is satisfied if $\mathbf{r}_i$ and $\mathbf{p}_i$ satisfy the Hamilton equations:

$$\frac{d\mathbf{r}_i}{dt} = \frac{\mathbf{p}_i}{m}, \quad \frac{d\mathbf{p}_i}{dt} = -\nabla U$$

Step 1: initiate the nuclei by random choice of "gaussians"
Choose their widths to fit experimental nuclear radius and binding energy.

$$\left(\sqrt{\langle r \rangle^2} - \sqrt{\langle r \rangle_{\text{exp}}^2}\right)^2 \quad \text{and} \quad \left(B - B_{\text{exp}}\right)^2$$

should minimize

System studied

$^{124}\text{Sn} + ^{124}\text{Sn}$ \hspace{1cm} N/Z = 1.48

$^{112}\text{Sn} + ^{112}\text{Sn}$ \hspace{1cm} N/Z = 1.24

$^{112}\text{Sn} + ^{124}\text{Sn}$ \hspace{1cm} $^{124}\text{Sn} + ^{112}\text{Sn}$

E/A = 50MeV \hspace{1cm} b = 0 \text{ and } 6\text{fm}
Introduction to Fluctuations

We want to introduce the density fluctuations having correct statistical properties near equilibrium, therefore we need to evaluate variances of density in space cells of a given volume. We can then randomly redistribute the particles according to these variances.

Assume a free Fermi quantum gas. Divide the system into cells of volume \( V \) in the co-ordinate space. Variance of the number of particles \( N \) in volume \( V \)

\[
\left\langle (\Delta N)^2 \right\rangle = T \left( \frac{\partial N}{\partial \mu} \right)_{T,V}
\]

(\( \mu \) - chemical potential) so the variance in density is:

\[
\left\langle (\Delta \rho)^2 \right\rangle = \frac{T}{V^2} \left( \frac{\partial N}{\partial \mu} \right)_{T,V}
\]
For a gas of free Fermions

\[ N = \frac{4V}{2\pi^2 \hbar^3} \int_0^\infty \frac{p^2 dp}{e^{(\varepsilon - \mu)/T} + 1} = \frac{4Vm^{3/2}}{\pi^2 \hbar^3 \sqrt{2}} \int_0^\infty \frac{\sqrt{\varepsilon}}{e^{(\varepsilon - \mu)/T} + 1} d\varepsilon \]

\( \varepsilon \) - energy, \( p \) - momentum, \( m \) - mass

so

\[ \sigma^2 = \left\langle (\Delta \rho)^2 \right\rangle = \frac{T}{V^2 \pi^2 \hbar^3 \sqrt{2}} \frac{\partial}{\partial \mu} \left[ \int_0^8 \frac{\sqrt{\varepsilon}}{e^{(\varepsilon - \mu)/T} + 1} d\varepsilon \right] \]

\[ \sigma^2 = \frac{16\pi m \sqrt{2m}}{V\hbar^3} \sqrt{\varepsilon_f T} \left( 1 - \frac{\pi^2 T^2}{12 \varepsilon_f^2} + \cdots \right) \]

\[ \varepsilon_f = \frac{\hbar^2}{2m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} \]

since

\[ \frac{\partial}{\partial \mu} \left( \frac{1}{e^{(\varepsilon - \mu)/T} + 1} \right) = -\frac{\partial}{\partial \varepsilon} \left( \frac{1}{e^{(\varepsilon - \mu)/T} + 1} \right) \]
\[ \sigma^2_\rho = -\frac{16}{Vh^3} \frac{\pi T m}{\sqrt{2m}} \sqrt{\epsilon} \frac{d}{d \epsilon} \left( \frac{1}{e^{(\epsilon-\mu)/T} + 1} \right) d\epsilon \]

and integrating by parts we get

\[ \sigma^2_\rho = \frac{8\pi T m \sqrt{2m}}{Vh^3} \int_0^\infty \frac{1}{\sqrt{\epsilon}} \left( \frac{1}{e^{(\epsilon-\mu)/T} + 1} \right) d\epsilon \]

Since an integral of the form

\[ I = \int_0^\infty \left( \frac{f(\epsilon)d\epsilon}{e^{(\epsilon-\mu)/T} + 1} \right) \]

can be written as

\[ I = \int_0^\mu f(\epsilon)d\epsilon + \frac{\pi^2}{6} T^2 f'(\mu) + \ldots \]

we get

\[ \sigma^2_\rho = \frac{8\pi T m \sqrt{2m}}{Vh^3} \left[ 2\sqrt{\mu} - \frac{\pi^2}{12} T^2 \frac{1}{\mu \sqrt{\mu}} + \ldots \right] \]
\[
\mu \approx \varepsilon_f \left[ 1 - \frac{\pi^2 T^2}{12 \varepsilon_F^2} \right]
\]

\[
\sigma^2 = \frac{16 \pi m \sqrt{2m}}{V \hbar^3} \sqrt{\varepsilon_f T} \left( 1 - \frac{\pi^2 T^2}{12 \varepsilon_F^2} + \ldots \ldots \right)
\]

with

\[
\varepsilon_F = \frac{\hbar^2}{2m} \left( \frac{3 \pi^2}{2} \rho \right)^{2/3} \frac{\hbar^2}{2m}
\]
What is needed to evaluate the variance of density fluctuations in a cell?

- local density $\rho$
- local temperature $T$

$\rho$ can be obtained by integrating

$$\rho(\vec{r}) = \int d^3 \vec{p} \ f(\vec{r}, \vec{p}, t)$$

$T$ can be obtained from the kinetic energy density

$$\overline{\varepsilon} = \frac{3}{5} \varepsilon_f \left( 1 + \frac{5\pi T^2}{12 \varepsilon_f^2} + \ldots \right)$$

The kinetic energy density must be calculated from non-collective motion only.

Simplest way of extracting the collective part of the momentum:

$$\overline{\vec{p}} = \frac{1}{N_{_{\text{overcell}}}} \sum_{i} \overline{\vec{p}}_i \quad \text{(average momentum)}$$

$$E_{_{\text{coll}}} = \frac{N_g}{N_t} \frac{\overline{\vec{p}}^2}{2m} \quad \text{(collective energy in cell)}$$

($N_g$ - per cell, $N_t$ - per nucleon)
124Sn+124Sn

50AMeV, semi-central

STOCHASTIC MEAN-FIELD

Time-scale matching:
Instability growth vs Interaction time

Rise and Fall:
-with impact parameter
- with beam energy

Freeze-out

V. Baran et al. NPA 703 (2002)
ISOSPIN TRANSPORT AT FERMI ENERGIES

$^{124}\text{Sn} + ^{112}\text{Sn}$ at 50 AMeV

- a) the neck region – low density interface
- b) pre-equilibrium particle emission

Stochastic BNV - transport model

- $b=8\text{fm} \rightarrow 120\text{fm/c}$
- $b=9\text{ fm} \rightarrow 100\text{fm/c}$ contact time
- $b=10\text{fm} \rightarrow 80\text{fm/c}$
Fig. 2. $^{124}$Sn + $^{124}$Sn collision at $b = 2$ fm, asy-stiff EOS.
Fig. 4. $^{124}$Sn + $^{124}$Sn collision at $b = 2$ fm, asy-soft EOS.
Fig. 5. The same as Fig. 4, but \( \delta = 6 \) fm.
ISOSPIN DIFFUSION AT FERMI ENERGIES

$^{124}\text{Sn} + ^{112}\text{Sn}$ at 50 AMeV
- 400 events for each asy-eos and impact parameter
- selected the binary events for $b=8, 9, 10$ fm

FERMI ENERGIES - two particularities affecting isospin equilibration dynamics

- pre-equilibrium emission: competition with the nucleon transfer process
- presence of low density interface: a different isospin dynamics

$$F_{n,p} = - \nabla_r U_{n,p} = - \frac{\partial U_{n,p}}{\partial \rho} \nabla_r \rho - \frac{\partial U_{n,p}}{\partial I} \nabla_r I$$

$b=8$ fm

$$U_{n,p} = A \frac{\rho}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^\sigma + U_{n,p}^{\text{sym}}(\rho, I)$$
**ISOSPIN IMBALANCE RATIO**

\[
R_P = \frac{2I_{124}^M - I_{124}^P - I_{112}^P - I_{112}^T}{I_{124}^P - I_{124}^T - I_{112}^P - I_{112}^T}; \quad R_T = \frac{2I_{124}^M - I_{124}^T - I_{112}^T - I_{112}^P}{I_{124}^T - I_{124}^P - I_{112}^T - I_{112}^P}
\]

Asysoft: more efficient for concentration gradients + larger fast neutron emission
Asystiff: more efficient for density gradients + larger n-enrichment of the neck IMFs

Momentum Dependence: faster dynamics and smaller isodiffusion
ISOSPIN SHARING AT FERMI ENERGIES

Mass and charge conservation:

$^{124}\text{Sn} + ^{112}\text{Sn}$

$\begin{align*}
I^M_p &= \frac{A_{0p}}{A_p} \left( I_{0p} - \frac{A_{gP}}{A_{0p}} I_{gP} - \left( \frac{A_{PT}}{A_{0P}} I_{PT} - \frac{A_{TP}}{A_{0P}} I_{TP} \right) \right) \\
I^M_T &= \frac{A_{0T}}{A_T} \left( I_{0T} - \frac{A_{gT}}{A_{0T}} I_{gT} + \frac{A_{PT}}{A_{0T}} I_{PT} - \frac{A_{TP}}{A_{0T}} I_{TP} \right)
\end{align*}$

$\nabla U_{p,n} = \frac{\partial U_{p,n}}{\partial \rho} \nabla \rho + \frac{\partial U_{p,n}}{\partial I} \nabla I$

affected by the lower density interface

dependence on asy-EOS

$\frac{A_{gP}}{A_{PT}} = x > 1$

$\begin{align*}
I^M_p &\approx \frac{A_{0p}}{A_p} \left( I_{0p} - \frac{A_{PT}}{A_{0P}} (x I_{gP} + (I_{PT} - I_{TP})) \right) \\
I^M_T &\approx \frac{A_{0T}}{A_T} \left( I_{0T} - \frac{A_{PT}}{A_{0T}} (x I_{gT} - (I_{PT} - I_{TP})) \right)
\end{align*}$

$^{124}\text{Sn} + ^{124}\text{Sn}$

$\begin{align*}
I^M_{124-124} &= \frac{A_{124}}{A_{124}} \left( I_{124} - \frac{A_{gP}}{A_{124}} I_{gP} \right) = I^M_T
\end{align*}$

$^{112}\text{Sn} + ^{112}\text{Sn}$

$\begin{align*}
I^M_{112-112} &= \frac{A_{112}}{A_{112}} \left( I_{112} - \frac{A_{gP}}{A_{112}} I_{gP} \right) = I^M_T
\end{align*}$
ISOSPIN EQUILIBRATION:
PRE-EQUILIBRIUM EMISSION vs DIFFUSION

\[ I_{gP}^{\text{asysoft}} > I_{gP}^{\text{sup asystiff}} ; \]
\[ I_{PT}^{\text{asysoft}} < I_{PT}^{\text{sup asystiff}} ; \]
\[ I_{gT}^{\text{asysoft}} > I_{gT}^{\text{sup asystiff}} ; \]
\[ I_{TP}^{\text{asysoft}} < I_{TP}^{\text{sup asystiff}} ; \]

\( b=8\text{fm} \)

\( b=10\text{fm} \)

\( I(t) \) vs time(fm/c)

asysoft eos – solid lines and black symbols
superasystiff eos – dashed lines and white symbols

AVERAGE over binary events
PRE-EQUILIBRIUM EMISSION EFFECTS

$I_{p}^{app}, I_{c}^{app}, I_{T}^{app}$ - the apparent asymmetry of the Projectile, of Composed system and of Target as a result of ONLY pre-equilibrium emission

$I_{p}, I_{T}$ - the real projectile, target asymmetries at freeze-out (as combined effect of diffusion and nucleon emission)

asysoft cos  — solid lines and black symbols
superasystiff eos — dashed lines and white symbols
FIG. 5: $^{124}$Sn + $^{112}$Sn collision with asysoft (top) and asysuperstiff (bottom) EOS as a function of collision time or impact parameter. Left panels: $I_P$ (black circles), $I_{gP}$ (crosses), $I_{PT}$ (rhombs) and $I_{TPT}$ (squares). Right windows: $I_T$ (black circles), $I_{gT}$ (crosses), $I_{PT}$ (rhombs) and $I_{TPT}$ (squares). The horizontal solid lines refer to the initial asymmetry of the projectile (target).
**ISOSPIN TRANSPORT: DRIFT + DIFFUSION**

Neutron/proton currents:

\[
\begin{align*}
\dot{j}_n &= -ct\nabla \mu_n(\rho_p, \rho_n, T) = -ct\left[\left(\frac{\partial \mu_n}{\partial \rho_n}\right)_{\rho_p,T} \nabla \rho_n + \left(\frac{\partial \mu_n}{\partial \rho_p}\right)_{\rho_n,T} \nabla \rho_p\right], \\
\dot{j}_p &= -ct\nabla \mu_p(\rho_p, \rho_n, T) = -ct\left[\left(\frac{\partial \mu_p}{\partial \rho_n}\right)_{\rho_p,T} \nabla \rho_n + \left(\frac{\partial \mu_p}{\partial \rho_p}\right)_{\rho_n,T} \nabla \rho_p\right],
\end{align*}
\]

with some algebra \((q=n,p)\)

\[
j_q = -D^\rho_q \nabla \rho - D^I_q \nabla I
\]

**Drift** → \(D^\rho_q \propto F(\rho) \pm 4I \frac{\partial C_{\text{sym}}}{\partial \rho}, (+n, -p)\)

**Diffusion** → \(D^I_q \propto C_{\text{sym}}\)

Isospin Transport:

1. Sensitivity to value and slope of the symmetry term
2. Paradox of Isospin Migration even in reactions between nuclei with the same charge asymmetry (due to density gradients in the reaction dynamics)
FIG. 6: Ratios of drift coefficients $R_q^i = \frac{D_q^{i,\text{asy},\text{superstiff}}}{D_q^{i,\text{asy},\text{soft}}}$, $i = \rho, I$ and $q = n, p$, as a function of the density for fixed asymmetry $I = 0.2$. 
ISOSPIN IMBALANCE RATIO: PRE-EQ. EFFECTS

\[ R_p = \frac{2I_P^M - I_P^{124} - I_P^{112} - I_P^{112}}{I_P^{124} - I_P^{112} - I_P^{112}} \]
\[ R_T = \frac{2I_T^M - I_T^{124} - I_T^{112} - I_T^{112}}{I_T^{124} - I_T^{112} - I_T^{112}} \]

approximations:
\[ \frac{A_{0P}^{124}}{A_P^M} \approx \frac{A_{0P}^{124}}{A_P^{124}}, \frac{A_{0P}^{112}}{A_P^{112}}, \frac{A_{0P}^{124}}{A_P^{124}}, \frac{A_{0P}^{112}}{A_P^{112}}, \frac{A_{0T}^{124}}{A_T^{124}}, A_{0T}^{112}, A_{0T}^{112} \]
\[ A_{0T}^{124} \approx A_{0T}^{112}, A_{0T}^{112}, A_{0T}^{112}, A_{0T}^{112}, A_{0T}^{112}, A_{0T}^{112} \]

Then:
\[ R_p = 1 - \frac{2A_{PT}^{124}(I_{PT} - I_{TP})}{I_{0P}^{124} - I_{0P}^{112} - A_{0P}^{124}(I_{gP} - I_{gf})} \]
\[ R_T = -1 + \frac{2A_{PT}^{112}(I_{PT} - I_{TP})}{I_{0T}^{124} - I_{112} - A_{112}^{124}(I_{gT} - I_{gf})} \]

\[ I_{TP}, I_{PT}, I_g \] Sensitive to asy-EOS
\[ I_{TP} - I_{TP}, I_{124} - I_{112} \] Less sensitive + compensation effects in \( R_{p,T} \)

asysoft faster equilibration – pre-equilibrium emission effect?
Conclusions

We have studied isospin equilibration and its dependence on asy- stiffness of EOS in case of semi-peripheral collisions at Fermi energy. A low density interface—a neck—which develops between two residues controls proton and neutron currents. Transfer of isospin, which is driven not only by the difference in N/Z ratio but also by density gradients, is sensitive to the density dependence of the asymmetry term. The interplay between these processes leads to stronger equilibration for asy-soft EOS. A rapidly increasing asymmetry energy at subnormal densities which occurs in asy-superstiff case, seems to be in better agreement with experimental data.