LIGHT ION INTERACTIONS
OF CONCERN FOR HADRONTHERAPY

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RATIONALE

Study of heavy ion induced phenomena

beam characterization

- maximizing the Tumor Control Prob.
- minimizing the Normal Tissue Complication Prob.

shielding optimization

- crew health
- instrument performance

a comprehensive and reliable description of all naturally followed reaction paths is requested

MonteCarlo approach
Nuclear reactions *at few ten MeV/n* occur in the Spread-Out Bragg Peak region producing

- positron emitters *(in-beam PET therapy monitoring)*
- heavy residues with high RBE *(treatment planning optimization)*
- projectile’s fragments with longer residual range
• **The BME theory** describing the thermalization of the composite system formed in $A-A$ collisions at $E < 100 \text{ MeV/n}$

• **Its MonteCarlo implementation** (including projectile and target’s break-up) simulating the deexcitation process up to evaporation residue identification

  *IMF d.d. spectra* for $^{12}C + ^{27}Al$ and $^{27}Al + ^{12}C$ at $13 \text{ MeV/n}$

• **FLUKA**, an Interaction and Transport MonteCarlo code, for a physically sound description of shower propagation in any complex geometry

• **Applications**

  characterization of *ion beams used* in radiobiological experiments and cancer therapy
Calculation of preequilibrium for the composite nucleus

proton and neutron momentum spaces divided into bins

\[ \{(px, py, pz) : pz \in [p_{zi}, p_{zi} + \Delta p_z), \quad \varepsilon = (2m)^{-1} (p_x^2 + p_y^2 + p_z^2) \in \left[ \varepsilon_i, \varepsilon_i + \Delta \varepsilon \right) \} \]

\( (z \text{ is the beam direction}) \)

of volume \( 2\pi m \Delta \varepsilon \Delta p_z \)
The BME system

\[ N_i = n_i \cdot g_i \]

nucleon number

number of states in bin \( i \)

occupation probability

\[
\frac{d(n_i \pi g_i \pi)}{dt} = \sum_{jlm} \left[ \omega_{ij}^{\pi \pi} g_i^{\pi} n_i^{\pi} g_j^{\pi} n_j^{\pi} (1 - n_i^{\pi})(1 - n_j^{\pi}) \right. \\
- \omega_{ij}^{\pi \pi} g_i^{\pi} n_i^{\pi} g_j^{\pi} n_j^{\pi} (1 - n_i^{\pi})(1 - n_j^{\pi}) \left] + \sum_{jlm} \left[ \omega_{ij}^{\nu \nu} g_i^{\nu} n_i^{\nu} g_j^{\nu} n_j^{\nu} (1 - n_i^{\nu})(1 - n_j^{\nu}) \right. \\
- \omega_{ij}^{\nu \nu} g_i^{\nu} n_i^{\nu} g_j^{\nu} n_j^{\nu} (1 - n_i^{\nu})(1 - n_j^{\nu}) \left] - n_i^{\pi} g_i^{\pi} \omega_{ij}^{\pi \pi'} g_{i'}^{\pi} \delta(\varepsilon_i^{\pi} - \varepsilon_{i'}^{\pi} - \varepsilon_{F}^{\pi} - B^{\pi}) - \frac{dD_i^{\pi}}{dt} \right. \]

\[ \sum_{jlm} \]
Multiplicity spectra

of emitted nucleons

\[
\frac{d^2 M(\varepsilon', \theta)}{d\varepsilon' d\Omega} = \frac{1}{2\pi \sin \theta} \int_0^{t_{eq}} n(\varepsilon, \theta, t) \frac{\sigma_{inv} v}{V} \rho(\varepsilon', \theta) \, dt
\]

of a cluster \(c\)

\[
\frac{d^2 M_c(E'_c, \theta_c)}{dE'_c d\Omega} = \frac{R_c}{2\pi \sin \theta} \int_0^{t_{eq}} N_c(E_c, \theta_c, t) \frac{\sigma_{inv,c} v_c}{V} \rho_c(E'_c, \theta_c) \, dt
\]

\[
N_c(E_c, \theta_c, t) = \prod_i \left( n_i^{\pi}(\varepsilon, \theta, t) \right)^{P_i(E_c, \theta_c) Z_c} \cdot \prod_i \left( n_i^{\nu}(\varepsilon, \theta, t) \right)^{P_i(E_c, \theta_c) N_c}
\]

joint probability
The BME theory is of limited applicability because

- it provides *inclusive spectra* (as averaged over many different preequilibrium paths)

- it gives a *mean equilibrated nucleus* (not $A_{CN}$, $Z_{CN}$, $E_{CN}^*$ distributions but their mean values)

- the compound nucleus turns out to **recoil always along $z$-axis**
  due to azimuthal symmetry of preequilibrium emissions
... to Monte Carlo code

\[
\int_{t}^{t+\Delta t} dt \int_{\theta}^{\theta+\Delta \theta} d\theta \int_{E}^{E+\Delta E} dE \frac{d^3 M_k(E, \theta, t)}{dE d\theta dt} \equiv p_k(E, \theta, t)
\]

\[(k \rightarrow \text{particle})\]

provided that \(\sum_k \int_{t}^{t+\Delta t} dt \int_{0}^{\pi} d\theta \int_{0}^{\infty} dE \frac{d^3 M_k}{dE d\theta dt} \lesssim 0.2\)

\( (\Delta t \text{ depends on } t) \)
Double differential neutron yield

$^{20}\text{Ne} + ^{165}\text{Ho} \ (E_{\text{Lab}} = 292\text{MeV})$

$^{20}\text{Ne} + ^{165}\text{Ho} \ (E_{\text{Lab}} = 402\text{MeV})$

Complete fusion residue excitation functions

$^{12}$C+$^{103}$Rh Excitation Functions

Energy (MeV)

Full circles: exp. data
Empty circles: theory
Projectile (or target) 's break-up

LPWBA

\[ T^{BF}(p_{sp.}) \propto |\hat{\phi}_{pa. sp.}(p_{sp.} - (m_{sp.}/m_P)p_P)|^2 \]

\( \hat{\phi}_{pa. sp.} \) is the wave function describing the relative motion of the two fragments inside the projectile

\( \vec{p}_P \) projectile momentum in the entrance channel

\( \vec{p}_{sp.} \) spectator momentum in the exit channel

\[ S(E_P, E_{sp.}, \vartheta_{sp.}) \propto p_{sp.} p_{pa.} |\hat{\phi}_{pa. sp.}(\vec{p})|^2 \]

Assuming an initial state interaction

\[ E_P' = E_P - E_L \]

with the probability for the energy loss \( E_L \)

given by

\[ P(E_L) = (K/C) \exp(-K(E_L - E_{min})) \]

where \( C = 1 - \exp(-K(E_P - E_{min})) \)

\[ \left[ \frac{d^2\sigma}{dE_{sp.}d\Omega_{sp.}} \right](E_P, E_{sp.}, \vartheta_{sp.}) = \sigma^{BF} \int_{E_{min}}^{E_P} P(E_L)S(E_P', E_{sp.}, \vartheta_{sp.})dE_L \]
IMF emission in light systems $[^7Be]$ 

$^{12}C + ^{27}Al$ @ 13 MeV/n 

$^{27}Al + ^{12}C$

nucleon coalescence $^{12}C$ break-up $^{27}Al$ break-up
IMF emission in light systems [B]

$^{12}C + ^{27}Al$

@ 13 MeV/n

$^{27}Al + ^{12}C$

nucleon coalescence

$^{12}C$ break-up

$^{27}Al$ break-up
IMF emission in light systems: the role of the evaporation

reaction mechanism

$^{23}\text{Na} + (^{4}\text{He} + ^{12}\text{C}) \rightarrow ^{23}\text{Na} + ^{16}\text{O}^*$

$^{22}\text{Ne} + (^{5}\text{Li} + ^{12}\text{C}) \rightarrow ^{22}\text{Ne} + ^{17}\text{F}^*$

$^{19}\text{F} + (^{8}\text{Be} + ^{12}\text{C}) \rightarrow ^{19}\text{F} + ^{20}\text{Ne}^*$

$^{16}\text{O} + (^{11}\text{B} + ^{12}\text{C}) \rightarrow ^{16}\text{O} + ^{23}\text{Na}^*$

$^{14}\text{N} + (^{13}\text{C} + ^{12}\text{C}) \rightarrow ^{14}\text{N} + ^{25}\text{Mg}^*$

$\rightarrow ^{39}\text{K}^*$

nucleon coalescence

$^{27}\text{Al}$ break-up

evaporation

$^{27}\text{Al} + ^{12}\text{C} \rightarrow ^{27}\text{Al}$

$\sigma$ [mb]

12

30

225
IMF emission in light systems \[ F' \]

\[ ^{12}C + ^{27}Al \]

@ 13 MeV/n

\[ ^{27}Al + ^{12}C \]

nucleon coalescence

\[ ^{27}Al \] break-up

evaporation
IMF emission in light systems \([ Na \)]

\[ ^{12}C + ^{27}Al \]

@ 13 MeV/n

\[ ^{27}Al + ^{12}C \]

\[ ^{27}Al \] break-up  evaporation
IMF emission in light systems [$Si$]

$^{12}C + ^{27}Al$ @ 13 MeV/n $^{27}Al + ^{12}C$

evaporation
FLUKA: generalities

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Interaction and Transport MonteCarlo code

Hadrons, leptons (incl. $\nu$), photons, heavy ions, low energy neutrons from thermal or few keV to cosmic ray energies

• Each component is treated as far as possible with the same accuracy
• All components in a single run, without intermediate steps
• FLUKA can be run in fully analog mode
  Its microscopic interaction models reproduce internal correlations
• It can also be run in biased mode
• Ion interactions based on DPMJET-III* (New!!)
  and a considerably improved version of rQMD-2.4†

http://www.fluka.org

Different applications [1]

The FLUKA development, its accuracy and versatility originated to a great deal from the needs of the author experiments, and new applications arise from new code capabilities, with a continuous interplay which is always physics driven. Examples are given below.

- **Neutrino physics and Cosmic Ray studies: initiated within ICARUS**
  - Neutrino physics: ICARUS, CNGS, NOMAD, CHORUS
  - Cosmic Rays: first 3D $\nu$ flux simulation, Bartol, MACRO, Notre-Dame, AMS

- **Accelerators and shielding: the very first FLUKA application field**
  - Beam-machine interactions: CERN, NLC, LCLS
  - Radiation Protection: CERN, INFN, SLAC, Rossendorf
  - Waste Management and environment: LEP dismantling, SLAC

- **Background and radiation damage in experiments: pioneering work for ATLAS**
  - all LHC experiments, NLC
Different applications [II]

- **Dosimetry, radiobiology and therapy**
  - Dose to Commercial Flights: E.U., NASA
  - Dosimetry: INFN, ENEA, GSF, NASA
  - Radiotherapy: already applied to real situations (Optis at PSI, Clatterbridge)
  - Dose and radiation damage to Space flights: NASA, ASI

- **Calorimetry**
  - ATLAS test beams
  - ICARUS

- **ADS, spallation sources (FLUKA+EA-MC, C.Rubbia et al.)**
  - Energy Amplifier
  - Waste transmutation with hybrid systems
  - Pivotal experiments on ADS (TARC, FEAT)
  - nTOF
Double differential neutron yield

Ar ions

Fe ions

400 MeV/n on thick Al targets

Isotopic distributions of fragmentation products

\[ ^{238}\text{U} + ^{208}\text{Pb} \ (750 \text{ A MeV}) \]

676 MeV/n $^{12}C$ beam on a water phantom

- Carbon ion intensity as a function of depth

- Build-up of boron ions as a function of depth
FLUKA - 1 GeV/n Fe “perfect beam” on PMMA

Fragment spectra as a function of depth in PMMA (nuclei only, 3° cone)

Fragment spectra at 19 g/cm² in PMMA for all charged, nuclei only, and with or without a 3° cut
FLUKA - 1 GeV/n Fe “perfect beam” on PMMA

**Track-average LET in PMMA**

- all charged
- nuclei
- nuclei, 3°
- LET threshold
- LET thr., 3°

**Dose average LET in PMMA**

- all charged
- nuclei
- LET threshold
- nuclei, 3° cone

Track-average LET as a function of depth in PMMA

(“true”, nuclei only, restricted to a 3° cone, with and w/o a cut corresponding to $e^-$ elimination)

Dose-average LET as a function of depth in PMMA

(“true”, with and w/o a cut roughly corresponding to $e^-$ elimination, nuclei only with and w/o a 3° cone restriction)
Characterization of therapeutic beams

OPTIS facility at PSI for eye tumors

72 MeV protons

M. Biaggi et al., NIM B 159, 89 (1999)
CONCLUSIONS AND PERSPECTIVES

modeling

*heavy ion interactions at low energies*

- The **BME theory** can give a reliable account of **preequilibrium emissions**, including **IMF's**

- Its **MonteCarlo implementation** permits to calculate exclusive cross sections and a wide variety of observables (excitation functions, recoil range and angular distributions of individual residues and double differential spectra of light and intermediate mass particles)

- Its coupling with **FLUKA**, for a more proper description of $A$–$A$ interactions **below 100 MeV/n**, is on the way

applications

- **hadrontherapy** optimization of **treatment planning**

- **space radiation protection** design of **spacecraft shielding** (for GCR) and special sheltering (for SPE)
Reaction cross section: a semiclassical expression

\[
\sigma_R = 2\pi \int_0^{\infty} \left(1 - T(b)\right) b \, db \quad T(b) = \exp \left(-\int_{-\infty}^{\infty} Q(b, z) \, dz\right)
\]

\(b\) impact parameter \quad \(z\) beam direction

following P.J. Karol, Phys. Rev. C 11, 1203 (1975)

\[
\sigma_R = \pi \left(a_T^2 + a_P^2\right) \left(E_1(\chi) + \ln \chi + \gamma\right) \left(1 - V_C/E_{ch}\right)
\]

\[
E_1(\chi) = \int_\chi^{\infty} \left(e^{-u/u}\right) \, du \quad \chi = \left(\pi^2 P(\xi) \bar{\sigma}(E_{nucl}) \rho_T(0) \rho_P(0) a_T^3 a_P^3\right) / \left(a_T^2 + a_P^2\right)
\]

\[
\bar{\sigma}(E_{nucl}) = \left[\left(\frac{Z_T}{A_T}\right) \left(\frac{Z_P}{A_P}\right) + \left(\frac{N_T}{A_T}\right) \left(\frac{N_P}{A_P}\right)\right] \sigma_{free}^{pp(nn)}(E_{nucl}) + \left[\left(\frac{Z_T}{A_T}\right) \left(\frac{N_P}{A_P}\right) + \left(\frac{Z_P}{A_P}\right) \left(\frac{N_T}{A_T}\right)\right] \sigma_{np}^{free}(E_{nucl})
\]

\[
a = a(R)
\]

\[
\rho(0) = (1/2) \rho_0 \exp\left((R/a)^2\right) \quad \rho_0 = 0.17 \text{ fm}^{-3}
\]

\[
\xi = \frac{E_F}{E_{nucl} + V}
\]

\[
\bar{E}_F(E_{nucl}) = B \exp(-KE_{nucl}) + C
\]

\[
V = \bar{E}_F + \left(S_T^T + S_n^T + S_P^P + S_n^P\right) / 4
\]
Reaction cross section: results
Fragment charge cross sections

\textbf{Fe 1.05 GeV/nucleon on Al}

\begin{center}
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=0.4\textwidth,
xlabel={Z},
ylabel={\(\sigma\) (mb)},
]
\addplot[only marks, mark options={scale=0.5}] table {data1.dat};
\addlegendentry{Zeitlin et al}
\addlegendentry{Cummings et al}
\addlegendentry{FLUKA}
\end{axis}
\end{tikzpicture}
\end{center}

\textbf{Fe 1.05 GeV/nucleon on Cu}

\begin{center}
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=0.4\textwidth,
xlabel={Z},
ylabel={\(\sigma\) (mb)},
]
\addplot[only marks, mark options={scale=0.5}] table {data2.dat};
\addlegendentry{Zeitlin et al}
\addlegendentry{Cummings et al}
\addlegendentry{Westfall et al}
\addlegendentry{FLUKA}
\end{axis}
\end{tikzpicture}
\end{center}

Analysis of projectile-like residues

$^{238}\text{U}^{+\text{208}}\text{Pb (750 A MeV)}$

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**Yield [nuclei/pr]**

- **A**
  - 0 to 250
  - $10^{-4}$ to 1

- **Z**
  - 0 to 100
  - $10^{-1}$ to 1

---

**Yield [nuclei/pr/GeV]**

- $E_{\text{exc}}$ [GeV]
  - 0 to 4
  - $10^{-2}$ to 1

- $\theta$ [°]
  - 175 to 180
  - 0 to 2