In the last few years a number of models based on simple circuital representations have been proposed to account for the resistive switching (RS) current–voltage (I–V) characteristics of metal–insulator–metal (MIM) structures. These devices typically exhibit two well-defined conduction levels after electroforming often referred to as the low and high resistance states that can be cyclically reached by the application of bipolar periodic voltage or current. The resulting hysteretic behavior arises from a reversible change of the electron transmission properties of the insulating film driven by an external stimulus. In this paper, after an overview of a variety of RS model proposals relying on circuital descriptions and basic analytic expressions, a model based on the solution of the generalized diode equation is discussed. The model is simple and flexible and consists of two opposite-biased diodes with series and shunt resistances that represent the filamentary current pathway spanning the oxide layer as well as the possible parasitic effects. The model parameters are governed by a mathematical entity called the logistic hysteron that can be linked to the internal state equation of the so-called memristive systems. For illustrative purposes, the bipolar periodic voltage or current. The resulting hysteretic behavior arises from a reversible change of the electron transmission properties of the insulating film driven by an external stimulus. In this paper, after an overview of a variety of RS model proposals relying on circuital descriptions and basic analytic expressions, a model based on the solution of the generalized diode equation is discussed. The model is simple and flexible and consists of two opposite-biased diodes with series and shunt resistances that represent the filamentary current pathway spanning the oxide layer as well as the possible parasitic effects. The model parameters are governed by a mathematical entity called the logistic hysteron that can be linked to the internal state equation of the so-called memristive systems. For illustrative purposes, the switching I–V characteristics of TiO$_2$-based MIM structures electroformed with different current compliances are examined in detail using this approach. Experimental results on bipolar RS by other authors are also assessed within the same framework.

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operational principle of Conducting Bridge RAMs (CBRAM), also called Electrochemical Metallization Memories (ECM) or Programmable Metallization Cells (PCM), which are based on the relocation of metal ions within a solid electrolyte. The transitions HRS → LRS can be abrupt (digital RS) or gradual (analog RS) indicating the sudden or progressive opening/closing of multiple parallel leakage paths or the narrowing/widening of the cross-section area of a single filament [494–97]. This latest property (tunable resistance) has been suggested for multi-level or multi-bit storage systems [54,71,98,99]. In general, the switching occurs after reaching certain threshold voltages with the same (unipolar RS) or with opposite (bipolar RS) polarities and they are called the SET (HRS → LRS) and RESET (LRS → HRS) voltages (see Fig. 1). The particular features of the switching processes seem to be related not only to the properties of the dielectric material but also to the metal electrodes and forming conditions. In addition, current compliances (CC) are often applied during the SET process in order to limit the thermal effects caused by the current runaway [26,100,101]. On the other hand, Joule heating at the bottleneck of the filament has been implicated in the dissolution mechanism of the conducting bridges, mainly for unipolar RS [51]. The first oxide breakdown event upon the application of electrical stress is called electroforming and corresponds to the generation of a localized weak oxide region susceptible to subsequent microscopic changes. From the statistical viewpoint, the electroforming event is fully consistent with the percolation theory of dielectric breakdown [90]. After this, the application of increasing and decreasing voltage sweeps leads to a pinched hysteresis loop of the current vs. voltage curve of the device [104]. Briefly, memristive systems are two-terminal circuit elements characterized by two coupled equations: one for the I–V curve of the device and one for its internal state variable [105–107]. While in linear systems the first relationship expresses an Ohmic-type dependence, the second one is written as a time derivative in order to account for the previous history of the device. The I–V model can be extended to nonlinear devices satisfying \( I(V = 0) = 0 \) and the state variable can be multidimensional [108]. Importantly, the connection between memristors and RRAMs has not been exempt from criticism and is still matter of debate [109,110]. In addition, it has been widely recognized the correspondence of the HRS and LRS with the soft-breakdown (SBD) and hard-breakdown (HBD) conduction modes occurring in thin dielectric films [64,86]. Both the SBD and HBD modes have been thoroughly investigated in ultrathin (\( t_{ox} < 5 \text{ nm} \)) \( \text{SiO}_2 \) layers as well as in thicker high permittivity (high-\( \kappa \)) dielectrics used as gate insulators in MOSFET devices [111]. Physical models for the filamentary conduction in \( \text{SiO}_2 \) and high-\( \kappa \) films have been reviewed in Ref. [112].

However, in spite of the major technological advances and better understanding of the physics behind RS achieved during the last few years, simple and flexible analytic models able to account for the wide variety of switching I–V curves exhibited by different dielectric films and electrode materials are hard to find in literature. While some of the available approaches are difficult to implement in circuit simulators because of the complexity of the physical processes involved or the mathematical constraints in the model equations and its derivatives [15,107,113–117], other models only focus the attention on the HRS and LRS I–Vs separately, completely disregarding the gradual transition between both states that in many cases characterizes the RS phenomenon. Other approaches are exclusively aimed at describing the SET and RESET switching dynamics caused by the application of current or voltage pulses [118–120]. Many SPICE-oriented models for RS have been recently reported [93,106,107,121–125], but their ability to accurately represent not only the electron transport characteristics in different materials but also their specific memory properties when subjected to arbitrary input signals has been seriously questioned [126]. Since the publication of Strukov’s memristor model in 2008 [127], a number of simple approaches based on combinations of linear, nonlinear and rectifying devices have been developed to describe the bistable conduction characteristics of electroformed MIM devices. Some of them are reviewed in Section 2. In Section 3, a simple circuit model for the hysteretic I–V characteristics is extensively discussed. The proposed approach consists of two opposite-biased diodes with series and shunt resistances that represent the filamentary current pathway spanning the oxide layer and the possible parasitic effects. Earlier versions of this model were reported elsewhere [128,129]. It is worth pointing out that, in this work, the emphasis is on the representation of the I–V curves rather than on the physics foundations of the RS electron transport mechanisms. The objective is to achieve a flexible, continuous and derivable model expressible by means of analytic functions. In order to reproduce the hysteretic behavior caused by the application of bipolar periodic voltage, the diode parameters in our model are driven by a mathematical entity referred to as the logistic hysteron (see Fig. 2a). As it is shown in Section 3, this entity, in its simplest form, can be written as a state equation for memristive systems. The hysteron concept within the present context comes from the celebrated Preisach model for the hysteretic \( B–H \) curve of ferromagnetic materials [130]. The relay hysteron, which is the fundamental building block of the Preisach model, describes the instantaneous activation and deactivation of a memory element with threshold voltages \( \alpha \) and \( \beta \) (see Fig. 2b). Here, the suitability of a smoothed version of this mathematical tool is demonstrated. The extension of the proposed

**Fig. 1.** The two basic resistive switching schemes: (a) unipolar RS and (b) bipolar RS. CC is the current compliance limit. HRS and LRS correspond to the high and low resistance states, respectively.
approach to the case of arbitrary input signals involves the decomposition of the logistic hysteron in elementary relay hysterons and the introduction of an important tool in the synthesis of hysteretic systems, namely the $\alpha - \beta$ memory plane [131]. This issue will not be covered here.

This paper is organized as follows: in Section 2, several models which can describe the HRS and LRS $I-V$ curves in terms of simple circuitual representations or function-fit equations are reviewed. After this, in Section 3, our own approach for the RS $I-V$ characteristics based on the solution of the generalized diode equation and the logistic hysteron is discussed. In Section 4, the switching $I-V$ characteristics of TiO$_2$-based MIM structures electroformed with different current compliances are considered for demonstrative purposes. The model is also fitted to experimental results obtained by other authors. Finally, in Section 5, the conclusions of this work are presented.

### 2. Equivalent circuit models for RS

In this Section, different models proposed to deal with the bistable $I-V$ characteristic exhibited by electroformed thin dielectric films in MIM structures will be discussed. Given the large number of publications related to RS, it is worth pointing out that the model list reported below is by far not exhaustive. Models that are described by simple mathematical expressions or which admit a circuitual representation will be exclusively analyzed here. Models that rely on the numerical solution of differential equations [114,115,132], iterative calculations [113,133], Monte Carlo simulations [134,135] or which exclusively focus the attention on the HRS $I-V$ curve adopting conventional electron transport mechanisms in dielectrics (Poole-Frenkel conduction [46,5 7.63,70,90,136], Schottky emission [15,32,35,45,47,54,76–78,137], space charge limited conduction [23,26,28.56,68,138,139], trap assisted tunneling [41,65,90,140], etc.) are also deliberately omitted. For additional information, the reader is redirected to the excellent review papers on these subjects [1,2,4,5,141–144].

#### 2.1. Dopant-drift model

The dopant-drift memristor model for TiO$_2$ layers proposed by Strukov et al. [127] represents a breakthrough in the field of RS devices. Although Strukov’s model has been widely investigated, mainly from the theoretical viewpoint [28,123,145,146], it is seldom applied in its original form to practical RS cases. In this regard, the application of the complementary series resistors model often requires the introduction of additional constraints in the state equation in the form of window functions [1,105,147]. The two equations that describe this model are (see Fig. 3):

\[
V = [R_{\text{RS}} - R_{\text{HRS}}]x + R_{\text{HRS}}I
\]

(1)

\[
x = Kf(x,I)
\]

(2)

where $0 \leq x \leq 1$ is the state variable ($x = w/D$ in Fig. 3a), $\dot{x}$ its time derivative, $K$ a constant related to the drift velocity of the oxygen deficiencies, $R_{\text{RS}}$ the LRS resistance, $R_{\text{HRS}}$ the HRS resistance, and $f$ the window function ($f=1$ in Strukov’s model). In general, $f$ is chosen so as to comply with $f(0,I) = f(1,I) = 0$ to ensure no drift of the dopant front beyond the boundaries of the device. One important aspect of Strukov’s model is the absence of well-defined threshold voltages for the SET and RESET events. For a simple sinusoidal input signal the model yields a pinched hysteresis loop (see Fig. 3b). However, a recent study has revealed lacking of predictability of the models expressed by Eqs. (1) and (2) regardless of the considered window function [126].

#### 2.2. Local reduction–oxidation model

One of the earliest attempts to describe the HRS $\leftrightarrow$ LRS transitions of the filamentary current by means of an equivalent circuit comprising nonlinear devices was proposed by Szot et al. [75], who attributed the switching of the electrical resistance in electroformed single-crystalline SrTiO$_3$ layers to a change in the transmission properties of individual dislocations. According to Szot’s model the switching process is basically a consequence of
the local modulation of the oxygen content related to the self-doping capability of TMOs. As schematically illustrated in Fig. 4, RS is associated with an electrochemical closing and opening process of a single dislocation at the surface of the dielectric. The electrical behavior of such nanowire is represented by an equivalent circuit formed by a network of resistors and diodes. It is also assumed that the inner network of dislocations is characterized by a 3D orthogonal lattice of resistors connected to individually addressable elements which can reversibly change between a diode and a resistor. Multiple filaments (detected as jumps in the I–V curves) are also possible within this framework. Although no analytic expression for the I–V characteristic is provided, the authors suggest that their model bridges the gap between the electrical behavior on the level of the discrete filamentary elements and the macroscopic properties of the material. The proposed circuit scheme is the initial seed of the diode-like model discussed in Section 3.

2.3. Memristor–rectifier model

The idea of using an equivalent circuit approach has also been considered by Yang et al. [148]. In this case, the hysteretic I–V curves are modeled using a combination of memristors and rectifiers which in turn can be used to construct a family of electronically reconfigurable circuit elements [58]. From the physical viewpoint, the device behavior is explained in terms of the coupled electron and ion dynamics driven by the applied electric field. Studying the bipolar switching I–V characteristics of micro- and nanoscale TiO2 junction devices with Pt electrodes, the authors were able to demonstrate that the HRS → LRS transitions involve changes in the potential barrier heights at the Pt/TiO2 interfaces due to the drift of positively charged oxygen vacancies (see Fig. 5a). The modifications of these barriers alternately lead to Ohmic- or rectifying-type characteristics. In this model, the I–V curves are mathematically described by the phenomenological equation (Fig. 5b and c):

$$I = w^n \beta \sinh(\alpha V) + \chi(\exp(\gamma V) - 1)$$

in which the first term represents a flux-controlled memristor: \( \beta \sinh(\alpha V) \) is the approximation considered for LRS, which is related to electron tunneling through a thin insulating residual barrier. \( \alpha, \beta, \) and \( n \) are fitting constants and \( w \) is the state variable of the memristor. While for \( n = 1 \) the drift velocity of the oxygen vacancies is directly proportional to the electric field, in the general case, \( n \) is used as a free parameter in the model which can be adjusted to modify the switching behavior. The second term in Eq. (3) represents the rectifying HRS which in this case adopts the form of a diode. \( \chi \) and \( \gamma \) are fitting constants. According to the authors themselves, Eq. (3) was chosen more for its simplicity and ability to reproduce the I–V characteristics than as a detailed physics model. A thorough analysis based on Local Pressure-modulated Conductance Microscopy (LPMCM) revealed that a localized conducting channel was responsible for the electronic switching.

A modification of Yang’s model for WOx-based MIM devices which involves an exponential voltage dependence in the state equation was proposed by Chang et al. [149]:

Fig. 4. Schematic representation of the RS phenomenon in SrTiO3 associated with the drift of oxygen vacancies: (a) corresponds to Ohmic-type conduction (LRS) and (b) corresponds to diode-like conduction (HRS). The state of the system is changed by the application of a localized voltage on the material surface. The dislocations are represented by an orthogonal network of resistors and diodes.

Fig. 5. (a) Energy diagram of the metal/dielectric interface showing the effect of the accumulation/deficiency of oxygen. The Schottky barrier represents the rectifying junction and the resistance represents the collapse of this barrier and the consequent appearance of the Ohmic behavior. \( \Phi_b \) and \( \Phi_e \) are the barrier height and its width, respectively. (b) Equivalent circuit model formed by a rectifier in parallel with a memristive device. (c) Typical simulated RS I–V curve using this model.
$I(V) = w_7 \sinh(\beta V) + (1 - w)\gamma(1 - \exp(-\delta V))$  \hspace{1cm} (4)

$w = \lambda \sinh(\eta V)$  \hspace{1cm} (5)

where the state variable $w$ is initially associated with the length of the gap that separates the doped (conductive) and undoped (resistive) device regions and later with the lateral size of the conductive filament. $\alpha, \beta, \gamma, \delta, \lambda$, and $\eta$ are fitting constants. In this case, Eq. (4) expresses the contribution of two parallel conduction mechanisms. For $w = 0$, Schottky-barrier emission dominates, whereas for $w = 1$, tunneling is the main conduction mechanism. The hyperbolic sine in Eq. (5) reflects the exponential dependence of the velocity of the vacancy front with the applied field. Expression (5) can be extended (using two independent exponentials instead of sinh) to account for the asymmetrical behavior of the $I$–$V$ curve for positive and negative biases. The authors have also shown that the fittings of hysteresis loops can be improved by adding a diffusion term in Eq. (5). The LTSPICE code for Chang’s memristor model is also provided in Ref. [149].

2.4. Tunneling barrier model

Borghetti et al. [150] investigated the electrical transport characteristics of TiO$_2$-based MIM structures as a function of the temperature. A completely different behavior was reported after electroforming compared to the fresh device, which indicates the transition from area-distributed current flow to localized conduction. Again, two distinctive limiting behaviors were observed: Ohmic-like for the LRS and exponential for the HRS. This latest mode is attributed to tunneling through a potential barrier and the switching to voltage-induced changes in the oxygen vacancy concentration in the gap between the tip of the filamentary path and the adjacent metal contact (see Fig. 6). The nonlinear HRS $I$–$V$ characteristic is approximated with a diode-like exponential dependence $i(V) = i_0(e^{V/V_0} - 1)$, where $i_0$ and $V_0$ are two state-dependent parameters. The differential resistance of the system is given by the expression $dv/di = R_L + (i_0/i_0)(i/i_0 + 1)$, where $R_L$ is the series resistance corresponding to the electroformed metallic channel in series with the tunneling gap. The role of $R_L$ in the metallic–nonmetallic transition is also investigated as a function of the temperature. Similar arguments were used by Pickett et al. [31], but, in this case, Simmons’ tunneling expression for a rectangular potential barrier with image force effect was considered for the $I$–$V$ curves [151]. The width of the tunneling barrier $w$ is identified as the dominant state variable and not the barrier height. The model includes the electric field-induced barrier lowering effect typical of Schottky contacts as well as ideality factors larger than one due to the presence of interface states. The model $I$–$V$ characteristics nicely agree with the experimental data. A typical simulated curve is illustrated in Fig. 7b.

2.5. Schottky barrier model

Hur et al. [137] proposed an equivalent circuit model for the bipolar $I$–$V$ characteristics in Ta$_2$O$_5$/TaO$_x$ stacks. The circuit is illustrated in Fig. 7a. In this case the HRS $I$–$V$ curve exhibits a rectifying behavior for negative applied bias (see Fig. 7b). The model is based on the modulation of the Schottky barrier height caused by the drift of oxygen vacancies. The devices require electroforming so that a filamentary pathway for conduction is implicitly assumed. Bistable switching occurs as the conducting path is oxidized or reduced in the vicinity of the metal electrode-oxide layer interface. In the LRS, the doped Ta$_2$O$_5$ region close to the interface is represented by a resistor while for HRS, the resulting undoped Ta$_2$O$_5$ is represented by a diode and a series resistance. The drift of oxygen vacancies in and out of the region of interest leads to the following dynamical equation for the length of the doped region $w(t)$:

$w = \mu_w R_{ON} \frac{I(t)}{L^3}$

where $\mu_w$ is the mobility of the oxygen vacancies, $R_{ON}$ the LRS resistance, and $L$ the active oxide thickness. Remarkably, Eq. (6) resembles the logistic equation which is later invoked in Section 3 in connection with the generalized diode-like model. In this case, the product $w(t)L - w(t)$ corresponds to the window function. $w(t)$ can now be used to calculate the total resistance of the device $R(t)$ as in Strukov’s model. An additional constant series resistance $R_K$ for the TaO$_x$ region is also taken into account. Then, the $I$–$V$ characteristic is expressed by a Schottky barrier model with a variable resistance $R(t)$ as:

$I(t) = \begin{cases} I_0 \exp \left( \frac{q}{kT} \left[ V(t) - I(t)R(t) \right] \right) - 1 \quad \text{for } V > 0 \\ I_0 \quad \text{for } V < 0 \end{cases}$

where $I_0$ is the diode current amplitude, $T$ the temperature, $q$ the electron charge, $k$ the Boltzmann constant, and $\eta$ the ideality factor. The model includes the electric field-induced barrier lowering effect typical of Schottky contacts as well as ideality factors larger than one due to the presence of interface states. The model $I$–$V$ characteristics nicely agree with the experimental data. A typical simulated curve is illustrated in Fig. 7b.

2.6. Sinh($x$)-based conduction models

Many recently proposed models describe the HRS current in electroformed devices by means of an hyperbolic sine dependence with the applied voltage. Occasionally, this consideration has also been extended to LRS [75]. This voltage dependence ultimately leads to pinched hysteretic loops, symmetric and Ohmic-type $I$–$V$ curves for low applied biases, and exponential behavior of the conduction characteristics for large applied biases. In some cases, this choice has a physics foundation but in a vast majority this particular dependence is simply considered because it yields good fitting results [75,153]. To our knowledge, Simmons and Verderber were the first to use an hyperbolic sine dependence for the reversible memory phenomena in thin insulating films [154]. They proposed a model for the $I$–$V$ curve of the form:

$I(V) = K[V \sinh(kV)]$  \hspace{1cm} (8)

where $K$ and $k$ are two functions of the applied voltage. Expression (8) was ascribed to direct tunneling in between defect sites and the
transitions HRS ↔ LRS to the existence of a resonant discrete energy level within the insulator forbidden band. Guan et al. [125] have also implemented a SPICE compact model for RS which makes use of a \( \sinh(x) \)-based expression. Inspired by the association of RRAM operation with the conductive filament growth as a consequence of the movement of oxygen vacancies, the authors assumed the formation of a tunneling barrier of variable width (see Fig. 8). In their model, the change of the gap length \( g \) is associated with the probability for oxygen ions to overcome activation energy barriers following an Arrhenius-type law:

\[
g = v_0 \exp \left( \frac{-E_{a,m}}{kT} \right) \sinh \left( \frac{qV}{kT} \right) \quad g \geq g_{\text{min}}
\]

where \( v_0 \) is a velocity related to the attempt-to-escape frequency and \( E_{a,m} \sim 1.2 \text{ eV} \) is the activation energy (migration barrier) for vacancy generation (oxygen migration) in a SET (RESET) process. \( L \) is the thickness of the switching material and \( a \) is the hopping distance. \( V \) is the voltage applied across the cell and \( g_{\text{min}} \) is the minimum gap size at which the tip of the filament is considered to be in contact with the electrode. \( \gamma \) is a local enhancement factor that takes into account the polarization of the material and the non-uniform potential distribution across the device. Eq. (9) is derived from the Mott–Gurney ionic hopping current [93]. Assuming that the current exponentially depends on the tunneling distance and the field strength, the authors proposed that the RS \( I-V \) characteristic can be expressed as:

\[
I(g, V) = I_0 \exp \left( -\frac{g}{g_0} \right) \sinh \left( \frac{V}{V_0} \right)
\]

where \( I_0, g_0, \) and \( V_0 \) are fitting parameters. Both the linear and exponential regions of the \( I-V \) curves can be captured by Eq. (10) by a proper selection of \( V_0 \). Unfortunately, in Ref. [125], the complete macromodel is only used to represent the transient response of RRAMs and not the \( I-V \) curves. An interesting point of the model is the inclusion of the Joule heating effect in the formation of the gap and the cycle-to-cycle stochasticity in \( g \).

A memristor model which takes into account the asymmetry of the experimental RS \( I-V \) curves was proposed by Yakopcic et al. [155]. The model is shown to be suitable for linearly increasing and sinusoidal inputs and is expressed as:

\[
I(t) = \begin{cases} 
  a_1 x(t) \sinh[bV(t)] & V \geq 0 \\
  a_2 x(t) \sinh[bV(t)] & V < 0 
\end{cases}
\]

where \( a_1 \) and \( a_2 \) are the amplitude parameters and \( b \) is a parameter that controls the intensity of the threshold functions. The state variable \( x(t) \) determines the switching dynamics of the device and its derivative is given by the equation:

\[
\dot{x} = \eta g(V)f(x)
\]
where $g(V)$ is the function implementing the threshold behavior and $f(x)$ a window function. $\eta$ is a fitting constant. The factorization into functions of $V$ and $x$ expressed by Eq. (12) is later recovered in Section 3 in connection with the dynamical equation for the logistic hysterons.

2.7. Quantum point-contact model

The quantum point-contact (QPC) model for dielectric breakdown [108,112,156,157] provides a natural explanation for the hyperbolic sine dependence discussed in the previous point. According to this model, the current that flows through a filamentary pathway between metal electrodes (electron reservoirs) is governed by a tunneling barrier corresponding to the first quantized level associated with the confinement of the electron wavefunction. The height of this barrier determines the conduction mode: LRS for a wide constriction (low barrier) and HRS for a narrow constriction (high barrier). A similar argument holds in terms of the width of the barrier. If the barrier is represented by an inverted parabolic potential and the applied voltage drops symmetrically at both ends of the conductive bridge, the $I-V$ characteristic can be obtained from the finite-bias Landauer approach as [158]:

$$\frac{dI}{dV} = \frac{4e^2}{\hbar^2} \exp(-\alpha\phi) \sinh \left(\frac{\alpha e V}{2}\right)$$ (13)

where $e$ is the electron charge, $\hbar$ the Planck constant, $\alpha$ a constant proportional to the width of the barrier, and $\phi$ the potential barrier height measured from the equilibrium Fermi energy (see Fig. 9a). Expression (13) strongly resembles Guan’s model for the $I-V$ curve (see Eq. (10)), except that in this case the prefactor in the current and the slope of the curve are correlated. Remarkably, if the barrier width shrinks to zero in Eq. (13), $\alpha \to 0$ so that the current reads:

$$I(V) = \frac{2e^2}{\hbar} V$$ (14)

which corresponds to the electron transport characteristic of a monomode ballistic conductor (transmission probability $T = 1$ in Fig. 9b). $G_0 = 2e^2/h = (12.9 \text{ k}\Omega)^{-1}$ is the quantum conductance unit. In this way, the RS current magnitude is determined by the lateral size of the filament: the $I-V$ characteristic follows an exponential law for a narrow constriction and a linear dependence as a function of the applied voltage for a wide constriction. The same result is obtained as a function of the barrier height but this analysis requires the complete expression for the $I-V$ curve [112]. Although the QPC model does not make explicit reference to the ultimate cause behind the resistance change, the central idea is that it originates in a local atomic rearrangement driven by the external applied field. The idea that the RS filament electrically behaves as a nanowire with conductance values close to integer multiples of $G_0$ is supported by numerous experimental observations [53,55,61,159–163]. It is worth pointing out that preferred atomic configurations for the filamentary path instead of true conductance quantization can also explain this phenomenology. First principle studies carried out in monclinic and amorphous HfO$_2$ thin films, in which oxygen vacancy paths are created, revealed that even the shortest filaments (one or two atoms long) can sustain conductive channels that exhibit signs of conductance quantization [164].

Interestingly, Eq. (13) can be rewritten as:

$$I(V) = \frac{2e^2}{\hbar} \exp(-\alpha\phi) \left[\exp\left(\frac{\alpha e V}{2}\right) - 1\right] - \exp\left(\frac{\alpha e V}{2}\right)$$ (15)

which from a circuital viewpoint can be represented by two opposite-biased parallel diodes. This corresponds to the circuit representation used in Szot’s model for the HRS [75]. Moreover, Eq. (15) can be generalized to the case of asymmetric potential drops at the two ends of the filament as:

$$I(V) = \frac{2e^2}{\hbar} \exp(-\alpha\phi) \left[\exp(\alpha e V) - 1\right] - \exp(-\alpha e (1-\beta)V - 1)$$ (16)

with $0 \leq \beta < 1$ a new parameter. $\beta$ is the fraction of the applied voltage that drops on the source side of the constriction. Notice that for $\beta = 1$, Eq. (16) reduces to the standard form of a Schottky barrier emission model, the main difference being the fact that in the QPC model the constant $\alpha$ is related to the shape of the potential barrier rather than to the inverse of the thermal energy $kT$. According to the QPC model, the temperature dependence mainly resides in $\phi$ [165]. On the other hand, for $\beta = 0$, the rectifying term in Chang’s memristor model is obtained [149]. For $\phi = 0$, $\beta = 1$, and $\alpha = 1/kT$, Eq. (16) reduces to the mathematical expression of a recently proposed model by Wagenaar et al. for AgS$_2$–based CBRAM devices [166]. Expression (16) can also account for the two rectifying junctions model proposed by Quinteros et al. [49] taking $\beta(V \geq 0) = 0$ and $\beta(V < 0) = 1$. In this latest case, the two metal-dielectric interfaces are treated as back-to-back switchable diodes.
with series resistance. Notice that self-rectifying structures, such as those required for intrinsic selectorless memories, can be obtained by eliminating one of the parallel diodes [167]. For a complete review about nanoscale diodes and the role played by the interfaces see Ref. [168]. In Section 3, the generalized diode equation and the use of the logistic hysteron for modeling the RS I–V characteristics of electroformed devices will be discussed.

3. The generalized diode model for RS

As it was pointed out in Section 2, a central idea in the modeling of the RS behavior of MIM devices consists in considering two coupled equations, in which one of the equations expresses a relationship for the I–V curve and the other equation somehow represents the memory effect. This second equation is often expressed as a time derivative of an internal state variable, which can adopt different meanings: resistance change, variation of the doped length, modulation of the barrier width and/or height, etc. A large number of the models proposed to deal with the I–V characteristics in RS devices relies on an Ohmic-type dependence between I and V which simply arises from the linear equation of memristive systems as originally proposed by Chua and Kang [103]. Notice that this assumption does not mean that the I–V curve is linear (because of the state equation). However, as we have already mentioned in Section 2, the I–V model can be extended to nonlinear devices. Of course, this is out from the canonical formulation of memristive

![Fig. 11](image1.png)

**Fig. 11.** (a) Evolution of the RS effect in the Ω-space. The two end points of the segment line correspond to the low and high resistance stationary states. (b) Evolution of the model parameters according to the logistic hysteron (see Fig. 2a).

![Fig. 12](image2.png)

**Fig. 12.** Experimental I–V characteristics measured in Al/TiO2/Au structures after electroforming with different current compliances Icc. Current compliances of 1 mA and 0.2 mA were used to limit the degradation of the devices during the voltage sweeps.

![Fig. 13](image3.png)

**Fig. 13.** (a) Experimental (symbols) and model (solid lines) results for the I–V curves for three different current compliances values used in the electroforming stage. Notice that the curves are highly asymmetric. (b) Details of the curves shown in (a). Model parameter values: αmax = 2.1 V⁻¹, αmin = 5.4 V⁻¹, Rmin = 80 Ω, Rmax = 1500 Ω, Imin = 590 mA, Imax = 110 µA, 0.85 V ≤ V′ ≤ 1.9 V, −1.75 V ≤ V′ ≤ −0.54 V, 5 V⁻¹ ≤ r′ ≤ 7.06 V⁻¹, 7 V⁻¹ ≤ r′ ≤ 11 V⁻¹.
systems. In this Section, we will consider the particular case of two opposite-biased diodes in connection with the model developed in Section 2.7. However, in what follows, we will not restrict the amplitude of the diode current and the potential profile parameter to those strictly stipulated by the quantum approach. In addition, we will include additional parameters such as series ($R_S$) and parallel resistances ($R_{P1}$ and $R_{P2}$) which could represent inherent or parasitic conduction effects (see Fig. 10). As in previous models, $R_S$ may represent a remnant local potential barrier, while $R_{P1}$ and $R_{P2}$ may represent localized and area-distributed parallel leakage current paths, respectively. In its general form, the proposed model makes neither an specific reference to the ion migration effect in the dielectric material nor to the kind of electron transport (filamentary or homogeneous). Of course, the selection of a particular conduction mechanism will reduce the number of free parameters according to the physical constraints.

The $I$–$V$ characteristic for the circuit illustrated in Fig. 10 is given by the transcendental equation:

$$I = I_0 \{\exp[\alpha (V + 1/R_S G_{P1} - R_S)] - 1\} - I_0 \{\exp[-\alpha (V + 1/R_S G_{P2} - R_S)] - 1\} + (V - R_S) G_{P1} + V G_{P2} (1 + R_S G_{P1})$$

(17)

where $G_{P1} = 1/R_{P1}$ and $G_{P2} = 1/R_{P2}$. Notice that Eq. (17) satisfies $I(0) = 0$ as expected for an odd function. Assuming that one diode is operating at a time, i.e. neglecting the inverse saturation current, the solution of Eq. (17) reads [169]:

$$I(V) = \text{sgn}(V) \left\{ (x R_S)^{-1} W[2d R_d \exp(2d(|V| + I_0 R_S))] + d (G_{P1}|V| - I_0) + G_{P2}|V| \right\}$$

(18)

where $d = (1 + R_S G_{P1})^{-1}$, $\text{sgn}$ is the sign function and $|x|$ is the absolute value of $x$. $W$ is the Lambert function [170], that is the solution of the equation $we^w = x$. Notice that regardless of the model parameter values, a pinched $I$–$V$ curve ($I(V = 0) = 0$) is always obtained. Although Eq. (18) is strictly valid for constant parameter values, we extend it ad hoc to the case of voltage-dependent parameters. In order to achieve the hysteretic behavior we introduce the logistic hysteron defined as:

$$\dot{\alpha}(V) = \{1 + \exp[-r^a (|V| - V^b)]\}^{-1}$$

(19)

where $r^a$ is a threshold parameter that varies in between 0 and 1 as a function of the applied bias. The plus and minus signs refer to the SET and RESET processes, respectively, which in this work are assumed to occur at opposite polarities (bipolar $R_S$). The extension to unipolar $R_S$ is straightforward. The state of the system is now described by the vector $\Omega = (I_0, x, R_S)$, where $I_0$ is the diode current amplitude, $x$ the low-voltage logarithmic conductance of the diode and $R_S$ the series resistance. For the sake of simplicity, $R_{P1}$ and $R_{P2}$ are taken as constants but they can be included in the state vector as well. In order to represent the modification of the system state caused by
Fig. 17. Experimental and model I–V characteristics for the 150 µA current compliance curve. (a) Effect of the $R_p$ on the I–V curves. $R_{p1} = 10$ kΩ, 100 kΩ, 10 kΩ, 3.3 kΩ and (b) effect of the $R_p$ on the I–V curves. $R_{p2} = 10$ GΩ, 10 kΩ, 3.3 kΩ and experimental 100 µA.

the applied voltage, the following parametric equation for $\Omega$ is postulated:

$$\Omega = \Omega_{\text{min}} + \lambda(\Omega_{\text{max}} - \Omega_{\text{min}})$$  \hspace{1cm} (20)

where $\Omega_{\text{min}} = (I_{\text{in}, \text{min}}, x_{\text{in}, \text{min}}, R_{\text{in}, \text{min}})$ and $\Omega_{\text{max}} = (I_{\text{in}, \text{max}}, x_{\text{in}, \text{max}}, R_{\text{in}, \text{max}})$ are the end points, minimum and maximum, respectively, of the segment line defined by $\Omega$ (see Fig. 11a). Fig. 11b illustrates the hysteretic behavior of the model parameters. If necessary, some of these parameters may also be constant. It is worth pointing out that this approach allows modeling the transition between two exponential I–V curves (if the potential drop across $R_p$ is negligible with respect to the applied voltage) and the transition between an exponential I–V curve and an Ohmic-type one (if the potential drop across $R_p$ is comparable to the applied voltage). Moreover, notice that Eq. (19) can be expressed in the generic form:

$$\lambda(V) = \left\{ 1 + \left( \frac{1 - \lambda_0}{\lambda_0} \right) \exp[-rV] \right\}^{-1}$$  \hspace{1cm} (21)

where $\lambda_0 = \lambda(V = 0)$. Since Eq. (21) is the solution of the logistic equation, the state equation for $\lambda$ is given by the expression:

$$\frac{d\lambda}{dt} = V\lambda(1 - \lambda)$$  \hspace{1cm} (22)

Notice the similarity of Eq. (22) with Eq. (6) and the natural appearance of the window function $\lambda(1 - \lambda)$. In this case, a voltage driven mechanism for the SET and RESET processes was assumed in Eq. (19) but the same can be done with respect to the current. The hysteretic behavior provided by $\lambda$ can also be interpreted as a delay between the applied voltage and the change in the electron transmission properties of the oxide layer [98,105]. The derivative of the logistic hysterons $d\lambda/dV$ has been related to the statistical distribution of the SET and RESET voltages of the elementary domains in multiferroic BiFeO$_3$ films [171,172]. In the present case, the logistic $\text{hysteron}$ is the key element for modeling the switching behavior of the I–V curve between two stationary states, namely HRS and LRS, and this transition is described as a segment line in the $\Omega$-space. For an arbitrary input signal and for an arbitrary memory effect (for instance with or without the wipe-out property), the evolution of the system state can be represented as a trajectory in this space. Here a single $\text{hysteron}$ was assumed for illustrative purposes but a decomposition on elementary relay $\text{hysteron}$s is also possible. In this regard, the proposed approach is equivalent to the Preisach model for the $B$–$H$ curve of magnetic materials [130,131].

Fig. 18. Experimental I–V characteristics extracted from literature (symbols) and model results (solid lines). (a) MnO$_2$ [173]: $x_{\text{in}} = x_{\text{max}} = 3.5 \ V^{-1}$, $I_{\text{in}, \text{min}} = 11$ µA, $I_{\text{in}, \text{max}} = 110$ µA, $R_{\text{in}, \text{max}} = 1 \ \Omega$, $V' = 0.4 \ V$, $V'' = -0.4 \ V$, $r' = r'' = 20 \ V^{-1}$; (b) Al$_2$O$_3$ [174]: $x_{\text{in}} = 1.6 \ V^{-1}$, $x_{\text{max}} = 1.9 \ V^{-1}$, $I_{\text{in}, \text{min}} = 700$ nA, $I_{\text{in}, \text{max}} = 270$ µA, $R_{\text{in}, \text{max}} = 1 \ \Omega$, $R_{\text{in}, \text{min}} = 200 \ \Omega$, $V_1 = 2.3 \ V$, $V_2 = -2.5 \ V$, $r' = 85 \ V^{-1}$, $r'' = 10 \ V^{-1}$; (c) Cu$_2$O [175]: $x_{\text{in}} = x_{\text{max}} = 3.4 \ V^{-1}$, $I_{\text{in}, \text{max}} = 20$ µA, $I_{\text{in}, \text{min}} = 800$ µA, $R_{\text{in}, \text{max}} = 500 \ \Omega$, $V_1 = 0.8 \ V$, $V_2 = -0.7 \ V$, $r' = r'' = 7 \ V^{-1}$; (d) HfO$_2$ [176]: $x_{\text{in}} = x_{\text{max}} = 2.1 \ V^{-1}$, $R_{\text{in}, \text{min}} = 1 \ \Omega$, $I_{\text{in}, \text{min}} = 25$ µA, $I_{\text{in}, \text{max}} = 400$ µA, $V_1 = 0.6 \ V$, $V_2 = -0.6 \ V$, $r' = 6 \ V^{-1}$, $r'' = 100 \ V^{-1}$.}
4. Experimental and fitting results using the diode-like model

In the first part of this section, we apply the model discussed in Section 3 to electroformed TiO$_2$-based MIM structures fabricated in our labs. After this, we apply the same approach to measurements performed by other authors obtained in a variety of devices. The first set of devices corresponds to crossbar-patterned Al(50 nm)/TiO$_2$(50 nm)/Au(50 nm) structures. The oxide films were grown by reactive sputtering with a pressure of 20 mTorr and a power of 150 W at room temperature. The bottom and top electrodes were deposited by the thermal evaporation method. Devices with area of 100 $\mu$m$^2$ were electroformed at approximately −11 V with current compliances $I_{CC}$ of 50, 100, and 150 $\mu$A. After electroforming, the devices exhibited bipolar RS as shown in Fig. 12. Compliances of 1 $mA$ and 0.2 $mA$ are used during the SET process. Compliance of 1.2 $mA$ is used for the RESET curves in the case of $I_{CC}$ = 150 $\mu$A. Variability from cycle-to-cycle is clearly observed in Fig. 12. Further details about the fabrication process of the devices and electrical characterization can be found in Ref. [128].

Fig. 13a shows some selected experimental $I$–$V$ curves and model results using Eqs. (18)–(20). Notice that the $I$–$V$ curves are not symmetric with respect to the applied bias and that exhibit negative differential resistance regions for $V < 0$. The proposed model is able to capture these features using an asymmetrical logistic hysteron. Fig. 13b shows a detail of the RESET curves. In order to illustrate the effect of the parameters on the model curves, a reference $I$–$V$ characteristic ($I_{CC}$ = 150 $\mu$A) was selected. First, the roles played by the hysteron parameters $r$, $V_r$ and $V_0$ on the model curves are illustrated in Fig. 14a and b, respectively. While $r$ determines the RESET transition rate, the threshold voltage $V_r$ sets the voltage at which the transition LRS → HRS takes place. As illustrated in Fig. 15, the resistance window is determined by the transition amplitude $I_{max} - I_{min}$. Increasing these parameters increases the diode current both in the LRS and HRS. Importantly, $I_{max}$ and $I_{min}$ can also affect both the shape of the SET and RESET transitions. As expected, the logarithmic conductance $\alpha$ (see Fig. 16a) and the series resistance $R_S$ (see Fig. 16b) mainly affect the HRS and LRS curves, respectively. Importantly, $R_{max}$ in combination with a properly chosen $I_{max}$ can be used to linearize the LRS $I$–$V$ curve. Concerning the parallel resistances $R_{P1}$ and $R_{P2}$, these parameters mainly modify the low-current/low-voltage region of the curves introducing a positive deviation from the exponential behavior (see Fig. 17). Both shunt resistances yield similar results so that they cannot be easily distinguished by simply considering the experimental $I$–$V$ curves. For our own experimental curves, these parameters play a minor role, but in other cases they can be worth considering.

To conclude, Fig. 18 shows several experimental $I$–$V$ curves extracted from different published papers. The experimental data were fitted using Eqs. (18)–(20). First, Fig. 18a shows bipolar RS for a TiN/MnO$_2$/Pt device with a 120 $\mu$m diameter electrode [173]. A forming process was required to induce RS at $1.5$ V. In this case $R_{min}$ and $R_{max}$ are negligible so that both LRS and HRS are described by exponential curves. Fig. 18b shows the RS behavior of radio frequency sputtered Al$_2$O$_3$ thin films [174]. In Fig. 18c, RS in a Cu$_2$O thin film is illustrated [175]. Finally, in Fig. 18d, experimental and fitting results are shown for HfO$_2$-based electroformed MIM structures [176]. In this case, the SET and RESET events take place at negative and positive biases, respectively, so that a clockwise motion for the logistic hysteron is considered.

5. Conclusions

Modeling of the resistive switching effect in electroformed MIM devices is a hot topic in the area of nonvolatile memory devices. The issue is also of great interest for the device physics community in general because of the connections with the field of memristive systems. In spite of the large number of published papers on this topic, there is no consensus yet about what is the best and more convenient approach available for representing the bistable $I$–$V$ characteristic exhibited by such structures. In this work, several models based on simple circuitual descriptions and mathematical expressions were reviewed. We have also proposed a very simple model based on the combination of diodes and resistances. To achieve the hysteretic behavior a mathematical entity called the logistic hysteron was used. Here, the focus was put on the representation of the conduction characteristics of devices subjected to periodic voltage signals. The generalization of these ideas to arbitrary input signals and to the particular memory properties of different materials is in progress.

Acknowledgments

A part of the work has been performed in the project PANACHE, co-funded by grants from Spain (Project Nos. PCIN2013-076 and TEC2012-32305 of the Spanish Ministerio de Economía y Competitividad) and the ENIAC Joint Undertaking. Project TEC2012-32305 was co-funded by the EU under the FEDER program. The authors also acknowledge the support of the project PICT- “MeMoSat”, MINICITP, Argentina and the DURSI of the Generalitat de Catalunya under contract 2014SGR384. Jordi Suñé also acknowledges the ICREA Academia award. Pablo Levy is member of CONICET, Argentina.

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