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Fractal Weyl law for Linux Kernel architecture

L. Ermann[1], A.D. Chepelianskii[2], and D.L. Shepelyansky[1,a]

[1] Laboratoire de Physique Théorique (IRSAMC), Université de Toulouse, UPS-CNRS, 31062 Toulouse, France
[2] LPS, Université Paris-Sud, CNRS, UMR8502, 91405 Orsay, France

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Abstract. We study the properties of spectrum and eigenstates of the Google matrix of a directed network formed by the procedure calls in the Linux Kernel. Our results obtained for various versions of the Linux Kernel show that the spectrum is characterized by the fractal Weyl law established recently for systems of quantum chaotic scattering and the Perron-Frobenius operators of dynamical maps. The fractal Weyl exponent is found to be $\nu \approx 0.65$ that corresponds to the fractal dimension of the network $d \approx 1.3$. An independent computation of the fractal dimension by the cluster growing method, generalized for directed networks, gives a close value $d \approx 1.4$. The eigenmodes of the Google matrix of Linux Kernel are localized on certain principal nodes. We argue that the fractal Weyl law should be generic for directed networks with the fractal dimension $d < 2$.

1 Introduction

The celebrated Weyl law [1] determines the number of quantum states in a given energy interval of conservative Hamiltonian systems in a phase space dimension d as a function of the Planck constant $\hbar$

$$N_E \propto \hbar^{-\nu}, \quad \nu = d/2. \tag{1}$$

Recently this law has been generalized to the fractal Weyl law applicable to open quantum systems with complex energy spectrum of nonunitary quantum operators [2–12]. In this case the exponent ν is linked to the fractal dimension of invariant sets of classical non-escaping orbits. Such type of operators appear in physical problems of quantum chaotic scattering, open quantum maps, metastable quantum states in molecules and nuclei.

In addition to open quantum systems it has been shown that the fractal Weyl law is also valid for the Perron-Frobenius operators of classical dynamical systems [13] with absorption or dissipation [14]. A finite size matrix approximant $\mathbf{S}$ of such operators is efficiently constructed using the Ulam method [15] which divides the whole phase space on N cells of size ϵ. The eigenvalues λ_i of the Ulam matrix approximant $\mathbf{S}$ of the Perron-Frobenius operator are located in a complex plane of a circle with $|\lambda| \leq 1$. For two-dimensional (2D) maps we have the matrix size $N \propto 1/\epsilon^2$ so that a cell area ϵ^2 effectively plays a role of $\hbar$ and $N_\lambda \propto N^\nu$ where N_λ is a number of eigenvalues λ_i with $|\lambda| \leq |\lambda_i| \leq 1$. For dynamical strange attractors and 2D maps with absorption we have $\nu = d/2$ where d is a fractal dimension of a dynamical invariant set, e.g. a fractal dimension of a strange attractor [14]. For $\nu < 1$ the majority of eigenvalues have $|\lambda| \to 0$ for $N \to \infty$.

In fact the Ulam method applied to 1D and 2D maps naturally generates the Ulam networks [16,17] which properties can be rather similar to the properties of scale-free directed networks such as the world wide web (WWW). Such type of networks naturally appear in various fields of science (see e.g. [18–24]). Therefore we can expect that the fractal Weyl law can appear also for directed networks similar to those of the WWW. The properties of such networks are well characterized by the Google matrix $\mathbf{G}$ [25] constructed on the basis of directed links between nodes:

$$G_{ij} = \alpha S_{ij} + (1-\alpha)/N \tag{2}$$

where the matrix $\mathbf{S}$ is obtained by normalizing to unity all columns of the adjacency matrix, and replacing columns with zero elements by $1/N$, N being the network size [22]. In the WWW context, the damping parameter α describes the probability to jump to any node for a random surfer. For $0 < \alpha < 1$ the only eigenvector with $\lambda = 1$ is the PageRank vector which has non-negative components and plays an important role in ranking of the WWW sites [22,24,25]. The matrix $\mathbf{G}$ belongs to the class of Perron-Frobenius operators [22]. The spectrum of $\mathbf{G}$ for the WWW university networks was studied recently in [26,27] and it was shown that a significant fraction of eigenvalues is concentrated at $|\lambda| \to 0$ that is a characteristic property of spectra with the Weyl law. However, no direct evidence for the fractal Weyl law for the WWW was found there for the reasons we explain below.

[a] e-mail: dima@irsamc.ups-tlse.fr

It is interesting to note that many complex networks are self-similar and have fractal properties (see e.g. [28,29] and references therein). Thus it was shown that the fractal dimension of networks of WWW and actors is $d = 4.1, 6.3$ respectively [28]. It is argued that the origin of fractal architecture of network is linked to a strong effective repulsion between the most connected nodes [29]. Such a generic fractal structure of complex networks also indicates that under certain conditions the fractal Weyl law can appear in the spectrum of the Google matrix of such networks. However, it is also important to note that usually the fractal properties are investigated for undirected networks or by converting directed networks into undirected ones (see e.g. [28,29]). Here, on a concrete example of the Linux Kernel network we show that the directionality of the network places a crucial role so that the directed network has different fractal dimension compared to the converted undirected network with the same nodes. The importance of directionality is very clear from the spectrum of the Google matrix: the spectrum is in a complex plane for a directed network while for an undirected network the spectrum is on a real line.

The paper is composed as following: the specrum of the Google matrix is analyzed in Section 2, the fractal properties of the Linux Kernel network are considered in Section 3, the properties of Google matrix eigenstates are studied in Section 4 and discussion of the results is given in Section 5.

2 Spectrum of Google matrix

To check the validity of the fractal Weyl law for a certain type of scale-free networks we focus on the procedure call network (PCN) of the Linux Kernel software introduced and studied recently in [30]. In the PCN the directed links are formed by the calls between procedures and the Google matrix (2) is constructed in the same way as for the WWW. The results of [30] show that only about 1% of eigenvalues have $|\lambda| > 0.1$ that is by a factor 50 smaller compared to the university networks of similar size studied in [26,27]. This gives a significant indication on appearance of the fractal Weyl law in the PCN. To study the spectral properties of $\mathbf{G}$ we use the data for the Google matrix at $\alpha = 0.85$ obtained in [30]. Our results are not sensitive to α which affects the spectrum of $\mathbf{G}$ in a rather simple way [16,17,22].

The eigenvalues λ_i and the right eigenvectors $\psi_i(j)$ satisfy the equation $\sum_{j'} G_{jj'}\psi_i(j') = \lambda_i\psi_i(j)$. To obtain the spectrum of the PCN with large sizes N we used the Arnoldi method from ARPACK library [31] that allowed to find all eigenvalues with $|\lambda| > 0.1$ up to the maximum $N = 285\,509$. The complex spectrum of λ_i is shown in Figure 1 for three versions of Linux Kernel. With the increase of the version number and the size N certain characteristic structures appear in the distributions of $\lambda_i = |\lambda_i| \exp(i\varphi_i)$ in the complex plane. Thus, there are clearly visible lines at real axis and polar angles $\varphi = \pi/2, 2\pi/3, 4\pi/3, 3\pi/2$. The later are related to certain cycles in procedure calls, e.g. an eigenstate at $\lambda_i = 0.85 \exp(i2\pi/3)$ is located only on 6 nodes. In Figure 1 we also present the spectrum of the Google matrix $\mathbf{G}^*$ which is obtained by an inversion of link directions in the adjacency matrix of the PCN as it was proposed in [30]. It is characterized by a similar spectrum structure but a larger radial distribution of λ_i.

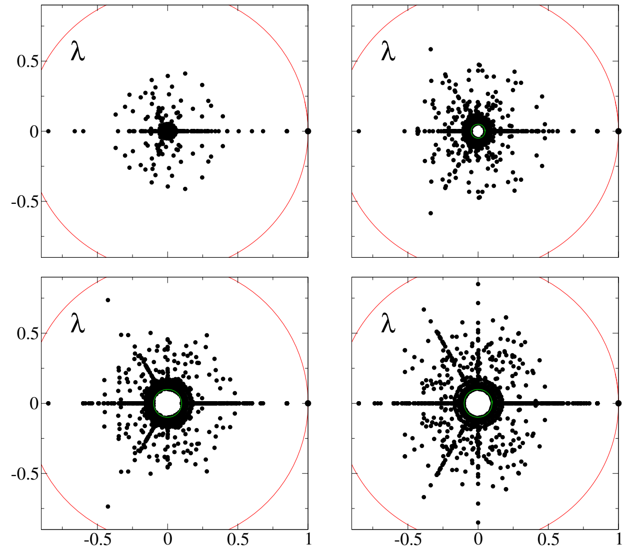

Fig. 1. (Color online) Distribution of eigenvalues λ in the complex plane for the Google matrix $\mathbf{G}$ of the Linux Kernel versions 2.0.40 (top left panel), 2.4.37.6 (top right panel) and 2.6.32 (left bottom panel) with $N = 14\,079$, $N = 85\,756$ and $N = 285\,509$ respectively. Bottom right panel shows eigenvalue distribution for the Google matrix $\mathbf{G}^*$ with inverted link directions in the PCN of the Linux Kernel version 2.6.32 with $N = 285\,509$. Solid lines represent the unit circle and the lowest limit of computed eigenvalues in top right and bottom panels.

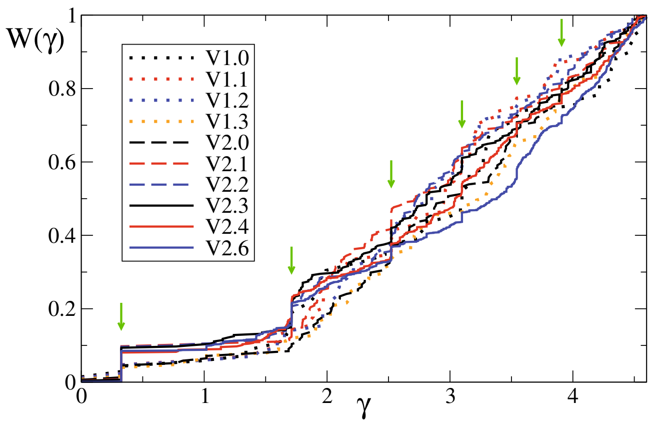

Fig. 2. (Color online) Dependence of the integrated density of states W on the relaxation rate $\gamma = -2\ln(\lambda)$ for the Google matrix of different versions of Linux Kernels from $V1.0$ with $N = 14\,079$ to 2.6.32 with $N = 285\,509$ as it is shown on the legend. The highly degenerate values of γ are marked by arrows at $\gamma_m = -2\ln(\alpha/m)$ for $m = 1, 2, 3, 4, 5, 6$.

The dependence of the normalized integrated density of states $W(\gamma)$, with eigenvalues $0 \le \gamma_i \le \gamma$, on the eigenvector relaxation rate γ is shown in Figure 2. Here

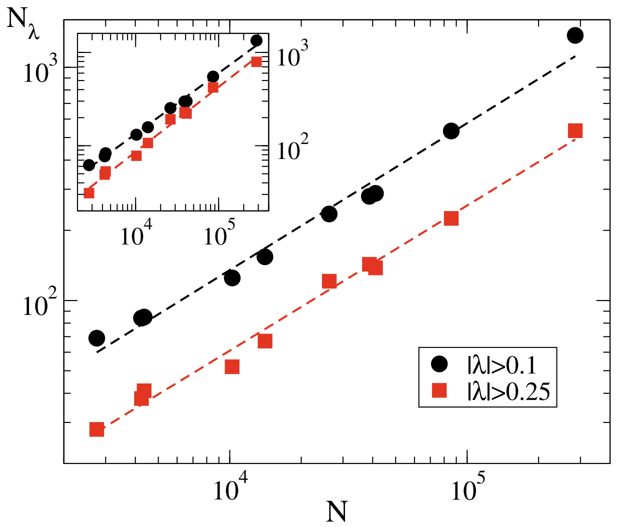

Fig. 3. (Color online) Dependence of the integrated number of eigenvalues N_λ with $|\lambda| > 0.25$ (red/gray squares) and $|\lambda| > 0.1$ (black circles) as a function of the total number of processes N for versions of Linux Kernels. The values of N correspond (in increasing order) to Linux Kernel versions 1.0, 1.1, 1.2, 1.3, 2.0, 2.1, 2.2, 2.3, 2.4 and 2.6. The power law $N_\lambda \propto N^\nu$ has fitted values $\nu_{|\lambda|>0.25} = 0.622 \pm 0.010$ and $\nu_{|\lambda|>0.1} = 0.630 \pm 0.015$. Inset shows data for the Google matrix $\mathbf{G}^*$ with inverse link directions, the corresponding exponents are $\nu^*_{|\lambda|>0.25} = 0.696 \pm 0.010$ and $\nu^*_{|\lambda|>0.1} = 0.652 \pm 0.007$.

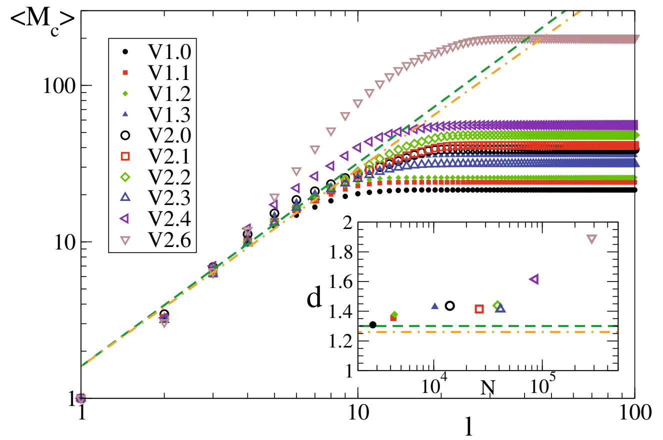

Fig. 4. (Color online) Cluster growing fractal dimension of the Linux Kernel directed networks. The average mass $\langle M_c \rangle$, with uniformly distributed seed, is shown for different Kernel versions as a function of the *network distance* l, in logarithmic scale. The corresponding fractal dimensions d for all versions are shown in the inset panel, where the power law fit is done for the range $1 \le l \le 10$. The dashed and dot-dashed lines (in both panels) represent the power law dependence $\langle M_c \rangle \propto l^d$, corresponding to fractal dimensions $d = 2\nu = 1.30$ and $d = 2\nu^* = 1.26$ from the fractal Weyl law of Figure 3 respectively.

$\gamma_i = -2\ln|\lambda_i|$ and $W(\gamma)$ is normalized on the number of states N_λ in the interval $\lambda \le |\lambda_i| \le 1$ with $\lambda = 0.1$. In average the dependence $W(\gamma)$ is the same for all 10 versions of Linux Kernel even if there are certain fluctuations. There are clear vertical steps in $W(\gamma)$ at $\gamma_m = -2\ln(\alpha/m)$ which are due to a presence of many degenerate states at the corresponding values of λ (e.g. 116 for $\lambda = \alpha$ for the version 2.6.32). Such degeneracy is known to be typical of the WWW networks (see e.g. [24,26,27]).

The network size N grows with the version number of Linux Kernel corresponding to its evolution in time. The dependence of N_λ on N, shown in Figure 3, clearly demonstrates the validity of the fractal Weyl law with the exponent $\nu \approx 0.63$ for $\mathbf{G}$ (we find $\nu^* \approx 0.65$ for $\mathbf{G}^*$). We take the values of ν for $\lambda = 0.1$ where the number of eigenvalues N_λ gives a better statistics. Withing statistical errors the value of ν is not sensitive to the cutoff value at small λ. The matrix $\mathbf{G}^*$ has slightly higher values of ν. These results show that the PCN of Linux Kernel has a fractal dimension $d = 2\nu \approx 1.26$ for $\mathbf{G}$ and $d = 2\nu \approx 1.3$ for $\mathbf{G}^*$.

3 Fractal dimension of Linux Kernel network

To check that the fractal dimension of the PCN indeed has this value we compute the dimension of the network by another direct method known as the cluster growing method (a description of this method can be find in [28]). In this method the average mass or number of nodes $\langle M_c \rangle$ is computed as a function of the *network distance* l counted from an initial seed node with further averaging over all seed nodes. In a dimension d the mass $\langle M_c \rangle$ should grow as $\langle M_c \rangle \propto l^d$ that allows to determine the value of d for a given network.

It is important to note that up to now the cluster growing method (see e.g. [28]) was used only for undirected networks. Our network is a directed one and, since we did not find relevant methods for such a case in a literature, we generalized this method in a simple way using the network distance l following only outgoing links. The average of $\langle M_c(l) \rangle$ is done over all nodes. Due to global averaging the method gives the same result for the matrix with inverted link direction (indeed, the total number of outgoing links is equal to the number of ingoing links). We will see that the fractal dimension obtained by this generalized method is very different from the case of converted undirected network.

The dependence of $\langle M_c \rangle$ on l, obtained for the directed network, is shown in Figure 4. Up to a saturation regime related to a finite number of nodes the growth of $\langle M_c \rangle$ is satisfactory described by a power law with a certain fractal dimension d. The straight lines, with the value of d obtained from the fractal Weyl law, give in average a satisfactory description of the growth $\langle M_c(l) \rangle$. More detailed data, with the fitted values of d for the interval $1 \le l \le 10$, are shown in the inset panel of Figure 4 for various network sizes N. For Linux versions up to V2.3 the fractal dimension remains in the interval $1.3 \le d < 1.45$ with an average $d \approx 1.4$. However, we have $d \approx 1.6$ for V2.4 and $d \approx 1.9$ for V2.6 versions. We attribute the strong deviation for the version V2.6 to the well known fact that significant rearrangements in the Linux Kernel have been

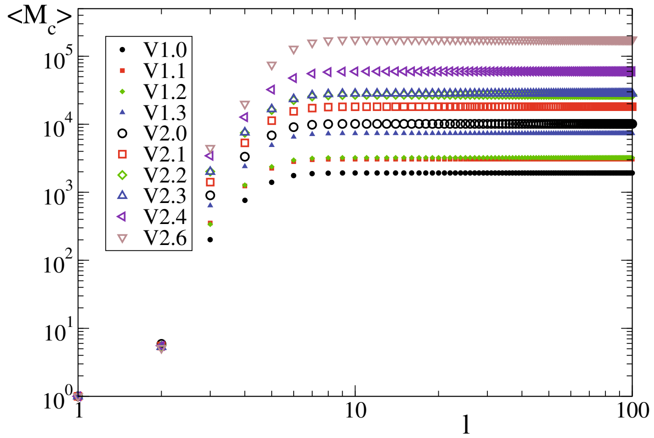

Fig. 5. (Color online) Cluster growing function for the undirected version of the Linux Kernel networks. The average mass $\langle M_c \rangle$, with uniformly distributed seed, is shown for different undirected Kernel versions as a function of the *network distance* l, in logarithmic scale.

done after version V2.4 [32,33]. Also the data of Figure 4 clearly show the saturation of $\langle M_c(l) \rangle$ at relatively small values $\langle M_c(l) \rangle < 200$ compared to the whole size of the network $N \leq 285\,509$. This happens due to existence of many isolated communities in the network which become linked only due to regularization of the Google matrix (1). This is the reason of strong degeneracy of the eigenvalue $\lambda = \alpha$ which has 116 values for the version V2.6. Also, in contrast to undirected networks, in directed networks there are nodes with only ingoing links that give an important reduction of the average mass.

Thus in view of the above restrictions we consider that there is a rather good agreement of the fractal dimension obtained from the fractal Weyl law with $d \approx 1.3$ and the value obtained with the cluster growing method which gives an average $d \approx 1.4$. The fact that d is approximately the same for all versions up to V2.4 means that the Linux Kernel is characterized by a self-similar fractal growth in time. The closeness of d to unity signifies that procedure calls are almost linearly ordered that corresponds to a good code organization. Of course, the fractal Weyl law gives the dimension d obtained during time evolution of the network. This dimension is not necessary the same as for a given version of the network of fixed size. However, one can expect that the growth goes in a self-similar way [21,29] and that static dimension is close to the dimension value emerging during the time evolution. This can be viewed as a some kind of ergodicity conjecture. Our data show that this conjecture works with a good accuracy up to the Linux Kernel V.2.6.

Here it is important to note that the properties of the directed network are very different from the converted undirected network with the same nodes. The later is obtained from the directed network by replacing all directed links by undirected ones. The results obtained by the clustering growing method for this undirected network are shown in Figure 5. We find that it is difficult to fit the dependence $\langle M_c(l) \rangle$ by a simple power law in this case. A very rough estimate gives very high value $d \sim 5$ being by a factor 4 larger than the dimension of the directed network. Also the saturation level of $\langle M_c \rangle$ is now comparable with the whole system size N while for the directed network it is smaller than N by three orders of magnitude.

Usually the methods of computation of fractal dimension have been developed for undirected networks (see e.g. [28,29,34,35]). Our example shows that the directed networks may have rather different characteristics compared to their undirected versions. Thus new methods for computation of fractal dimension in directed networks should be developed. Our results show that the generalized version of the method described in [28] work well in the case of directed networks.

4 Google matrix eigenstates

Before we discussed the properties of the spectrum of the Google matrix of the PCN. Let us now analyze the properties of its eigenstates $\psi_i(j)$. As it is usual for the disordered systems with the Anderson localization (see e.g. [36]), we use the participation ration (PAR) ξ to characterize the localization properties of the eigenstates. It is defined as $\xi_i = (\sum_j |\psi_i(j)|^2)^2 / \sum_j |\psi_i(j)|^4$. Physically, ξ_i gives the number of nodes effectively occupied by an eigenstate $\psi_i(j)$. The dependence of ξ on $|\lambda|$ for three versions of the Linux Kernel is shown in Figure 6. There is only a small increase of ξ with size N which is however much smaller than the increase of N from one version to another. Also even maximum values of ξ remain by a factor 5×10^3 smaller than the system size N. Thus we conclude that the eigenstates are essentially localized.

The dependence of probability of eigenstates, with largest values of $|\lambda_i|$, on the PageRank index j is shown in Figure 7 for the version 2.6.32. The main peaks of probability are scattered over the whole system size N. The appearance of vertical stripes is clearly visible. This signifies that statistically the positions of peaks are repeated from one eigenstate to another. It is possible that this effect is related with certain cycles between procedures in the Linux Kernel. These cycles, or communities as it is used to say for the WWW, appear even on a far tail of the PageRank. The origin of such weakly isolated cycles and correlations between different relaxation modes requires further detailed studies.

5 Discussion

Here we characterized the directed network by the Google matrix and investigated its spectrum and eigenstate properties. We think that such an approach gives much richer charcteristization of networks. Indeed, as it was shown in [26,27], the spectrum of the Google matrix depends in a sensible way on network structure so that networks with rather similar distribution of links may have very drastic

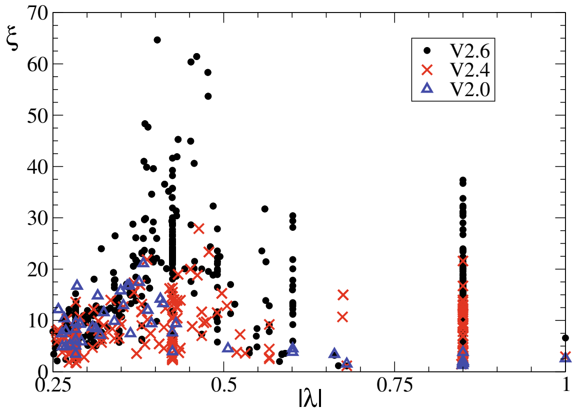

Fig. 6. (Color online) Localization properties of Google matrix eigenstates for Linux Kernel versions 2.0.40 2.4.37.6 and 2.6.32 with $N = 14\,079$, $857\,56$, $285\,509$ respectively. The participation ratio ξ of eigenstates is shown as a function of modulus of the corresponding eigenvalues $|\lambda|$; the values of $\langle\xi\rangle$, averaged over all given interval of $|\lambda|$, are $9.5, 8.6, 17.3$ respectively.

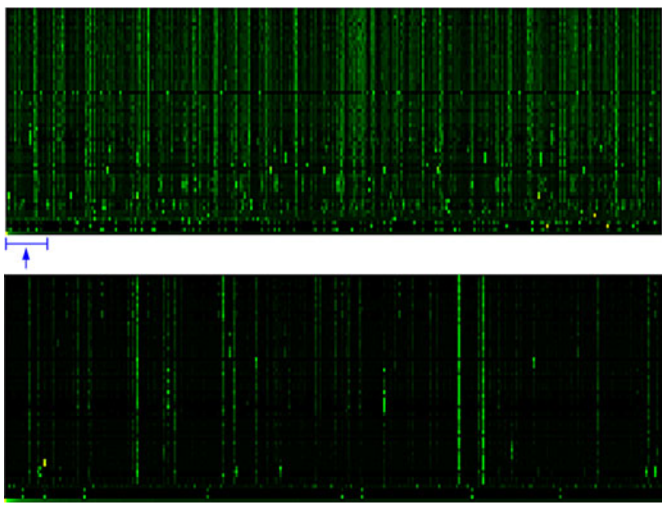

Fig. 7. (Color online) Coarse-grained probability distribution $|\psi_i(j))|^2$ for the eigenstates of the Google matrix for Linux Kernel version 2.6.32. The horizontal lines show the first 64 eigenvectors ordered vertically by $|\lambda_i|$ from $|\lambda| = 1$ on bottom to $|\lambda| \approx 0.4$ on top (only one state is shown for degenerate eigenvalues, all states are normalized by $\sum_j |\psi_i(j)|^2 = 1$, index j corresponds to the order given by the PageRank located at the bottom line). The top panel shows the coarse-grained complete network of 285 509 sites with 307 cells, each containing 930 sites; the bottom panel shows the coarse-grained distribution for first 300 cells, each containing 62 sites (bar and arrow on the top panel mark the region of bottom panel). Probability is proportional to color with black for zero and yellow/white for maximum.

differences in the spectrum (e.g. presence or absence of gap in a vicinity of $\lambda = 1$). We think that further studies of the Goggle matrix properties of complex networks will bring us deeper understanding of their generic properties.

In conclusion, our results show that the Google matrix of the PCN of the Linux Kernel is characterized by the fractal Weyl law with the fractal Weyl exponent $\nu \approx 0.65$. This value corresponds to the fractal dimension of the network $d \approx 1.3$. This dimension is smaller than two and thus the fractal Weyl law becomes well visible. In contrast to that for networks with the dimension $d \geq 2$ the fractal Weyl law is replaced by a usual dependence $N_\lambda \propto N$ since $\nu = d/2 > 1$. It is known that the WWW has the fractal dimension $d \approx 4.1$ [28] that explains why the fractal Weyl law does not work for the WWW university networks discussed in [26,27]. On the basis of our result we argue that the fractal Weyl law should appear in all directed networks with the fractal dimension $d < 2$. It is important to note that the fractal Weyl exponent ν is not sensitive to the exponent characterizing the decay of the PageRank: indeed, according to [30] the later remains the same for the WWW and the PCN of Linux Kernel while the values of ν are different. We think that the further studies of the fractal Weyl law for directed networks with $d < 2$ will bring us to a deeper understanding of their scale-free properties. Our results also show that the properties of directed networks can be rather different from their undirected versions.

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