QCD, Strings and Black holes

The large N limit of Field Theories and Gravity

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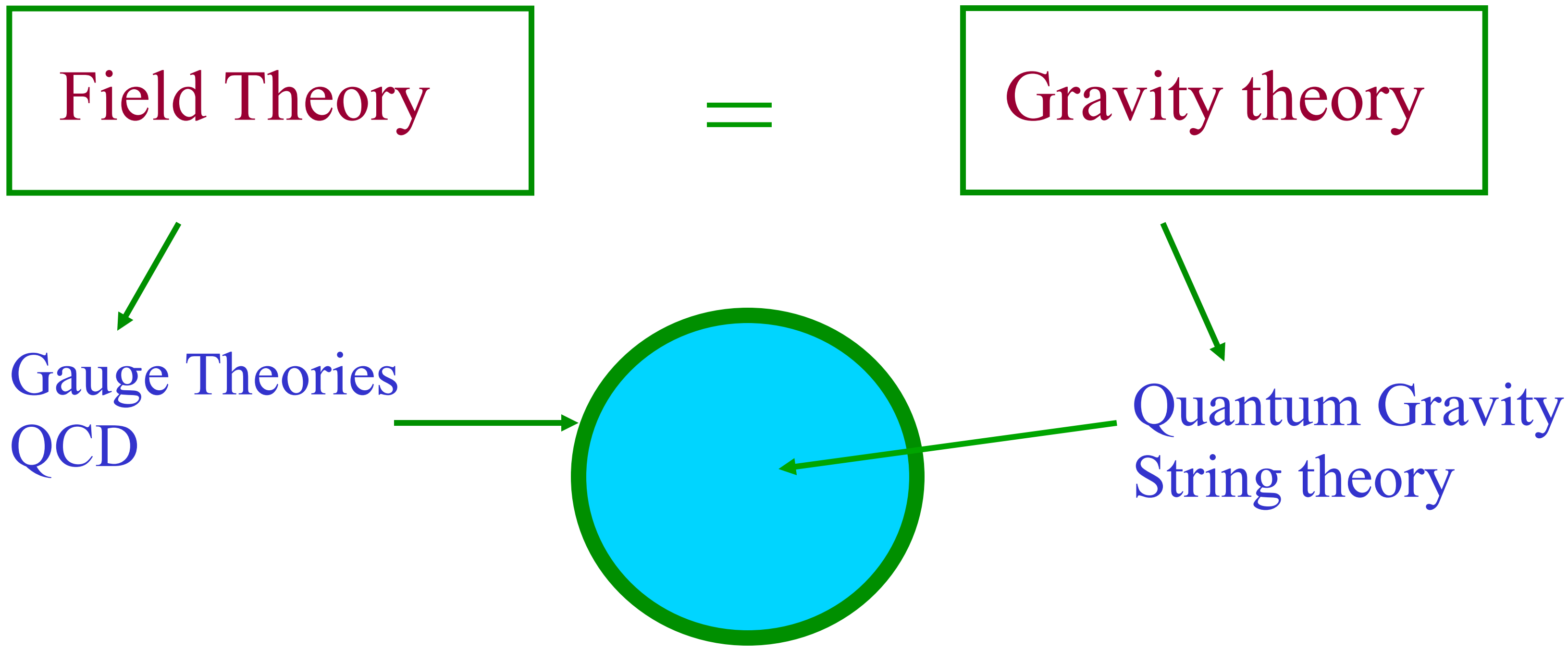


Plan

QCD, Strings, the large N limit

Supersymmetric QCD

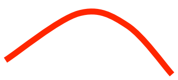
↓ N large

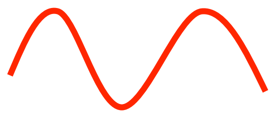
Gravitational theory in 10 dimensions

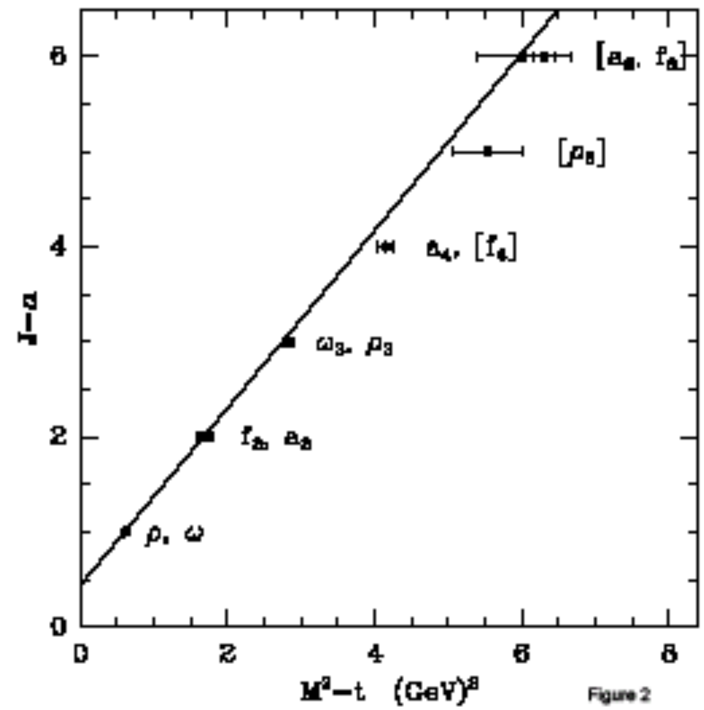
Black holes

Strings and Strong Interactions

Before 60s → proton, neutron → elementary ?
During 60s → many new strongly interacting particles
Many had higher spins s = 2, 3, 4 ….
All these particles → different oscillation modes of a string.

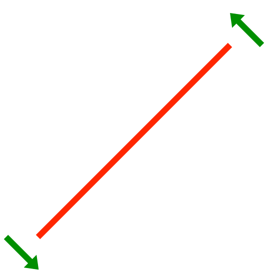






From E. Klempt **hep-ex/0101031**

This model explained features of the spectrum of mesons.



Rotating String model

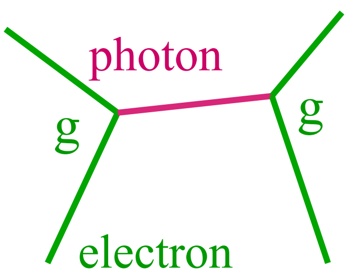
$$m^2 \sim TJ_{\max} + const$$

Strong Interactions from Quantum ChromoDynamics

Experiments at higher energies revealed quarks and gluons

- 3 colors (charges)
-
- They interact exchanging gluons

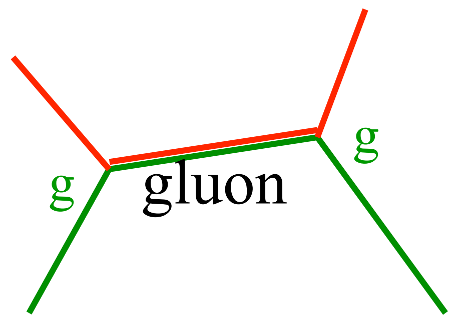
Electrodynamics



Gauge group

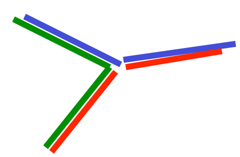
U(1)

Chromodynamics (QCD)



3 x 3 matrices

SU(3)



Gluons carry color charge, so they interact among themselves

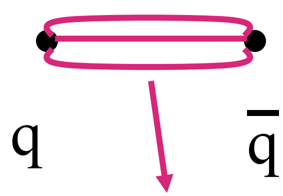
Coupling constant decreases at high energy

Gross, Politzer, Wilczek

$g \longrightarrow 0$ at high energies $\longrightarrow$ QCD is easier to study at high energies

Hard to study at low energies

Indeed, at low energies we expect to see confinement



Flux tubes of color field = glue

$$V = T\ L$$

At low energies we have something that looks like a string. There are approximate phenomenological models in terms of strings.

How do strings emerge from QCD?
Can we have an effective low energy theory in terms of strings ?

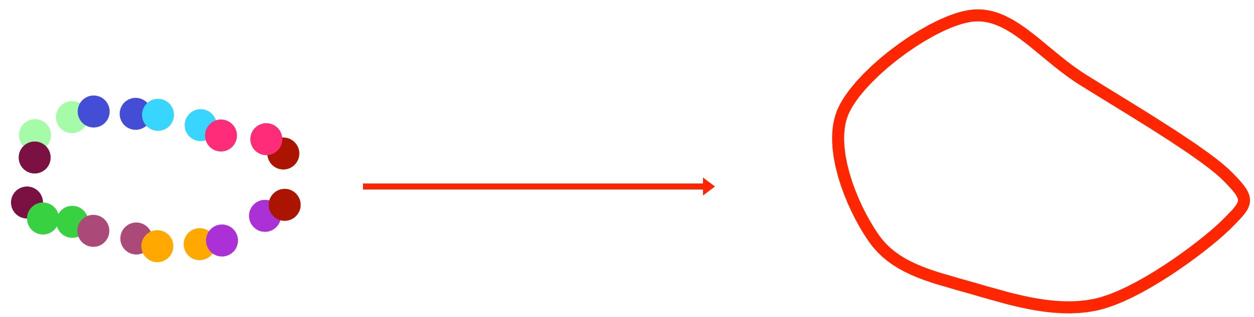
Large N and strings

Gluon: color and anti-color

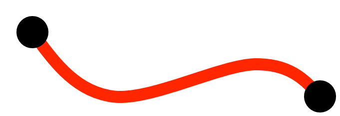
Take N colors instead of 3, SU(N)

t' Hooft '74

Large N limit



g^2N = effective interaction strength
when colors are correlated



Open strings → mesons
Closed strings → glueballs

General Idea

- Solve first the $N=\infty$ theory.

- Then do an expansion in $1/N$.

- Set $1/N = 1/3$ in that expansion.

The N=∞ case

- It is supposed to be a string theory
- Try to guess the correct string theory
- Two problems are encountered.

1. Simplest action = Area

Not consistent in D=4 (D=26 ?) Lovelace

↓ generate Polyakov

At least one more dimension (thickness)

2. Strings theories always contain a state with m=0, spin =2: a Graviton.

But:

- In QCD there are no massless particles.
- This particle has the interactions of gravity

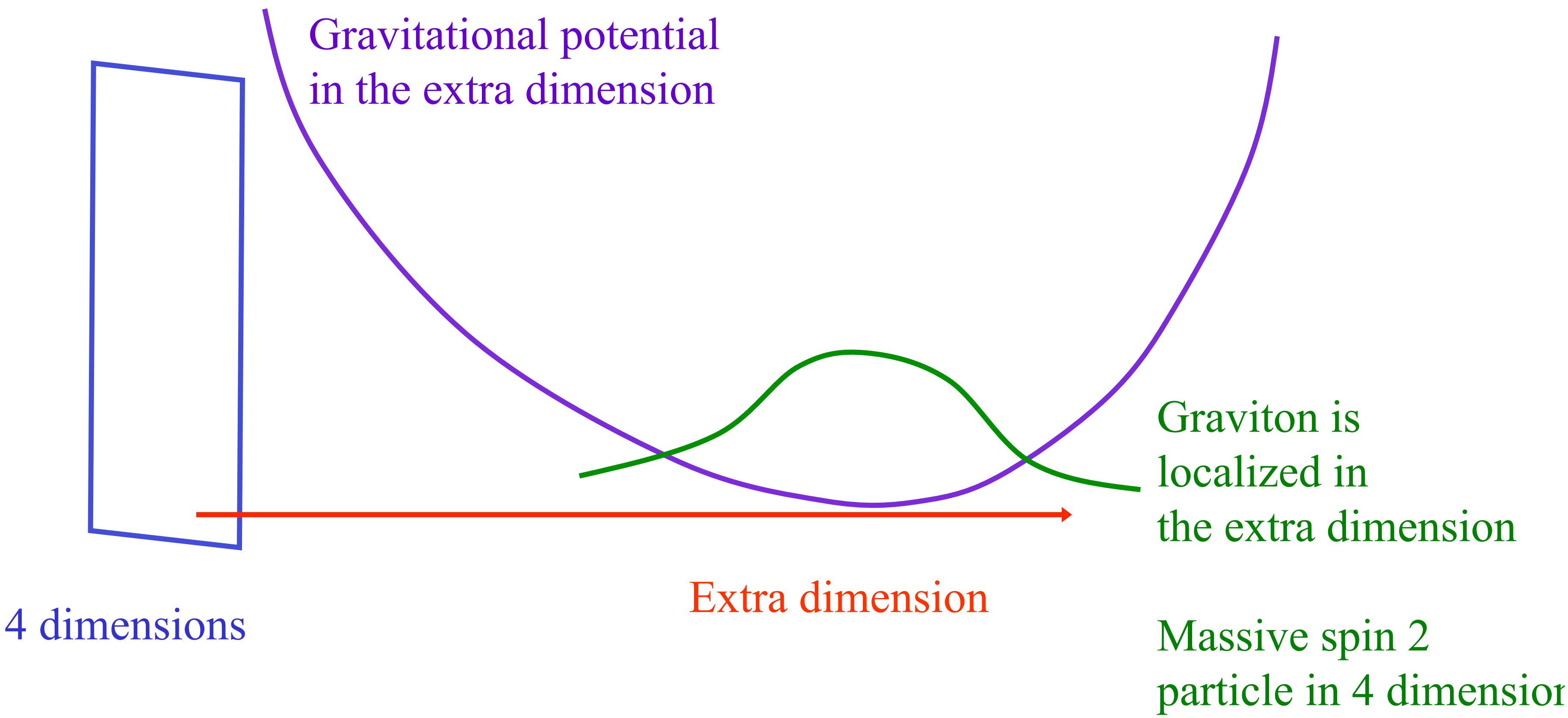
Scherk-Schwarz
Yoneya

For this reason strings are commonly used to study quantum gravity. Forget about QCD and use strings as a theory of quantum gravity. Superstring theory, unification, etc.

But what kind of string theory should describe QCD ?

We combine these two problems into a solution.

-We will look for a 5 dimensional theory that contains gravity.



We need to find the appropriate 5 dimensional geometry

It should solve the equations of string theory

They are a kind of extension of Einstein's equations

Very difficult so solve

Consider a simpler case first. A case with more symmetry.

We consider a version of QCD with more symmetries.

Most supersymmetric QCD

Supersymmetry

Ramond
Wess, Zumino
Volkov Akulov

Bosons ⟷ Fermions

Gluon ⟷ Gluino

Many supersymmetries

B1 ⟷ F1
B2 ⟷ F2

Maximum 4 supersymmetries, $N = 4$ Super Yang Mills

Susy might be present in the real world but spontaneously broken at low energies.
So it is interesting in its own right to understand supersymmetric theories.

We study this case because it is simpler.

Similar in spirit to QCD

Difference: most SUSY QCD is scale invariant

Classical electromagnetism is scale invariant

$$V = 1/r$$

QCD is scale invariant classically but not quantum mechanically, g(E)

Most susy QCD is scale invariant even quantum mechanically

<u>Symmetry group:</u>

Lorentz + translations + scale transformations + other

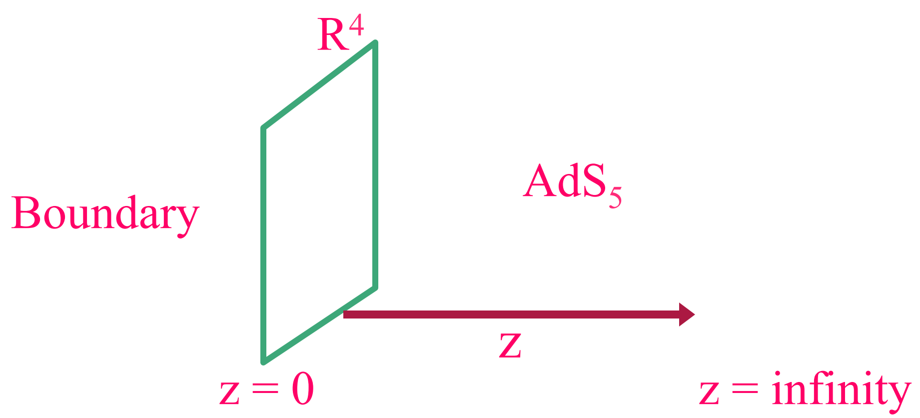
These symmetries constrain the shape of the five dimensional space.

$$ds^2 = R^2 \; w^2(z) \; (dx^2_{3+1} + dz^2)$$

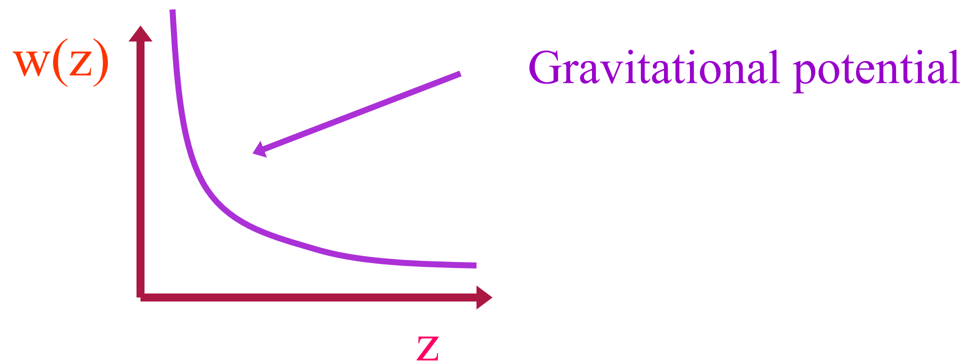
↓ redshift factor = warp factor ~ gravitational potential

Demanding that the metric is symmetric under scale transformations $x \rightarrow \lambda x$, we find that $w(z) = 1/z$

$$ds^2 = R^2 \frac{(dx^2_{3+1} + dz^2)}{z^2}$$



This metric is called anti-de-sitter space. It has constant negative curvature, with a radius of curvature given by R.

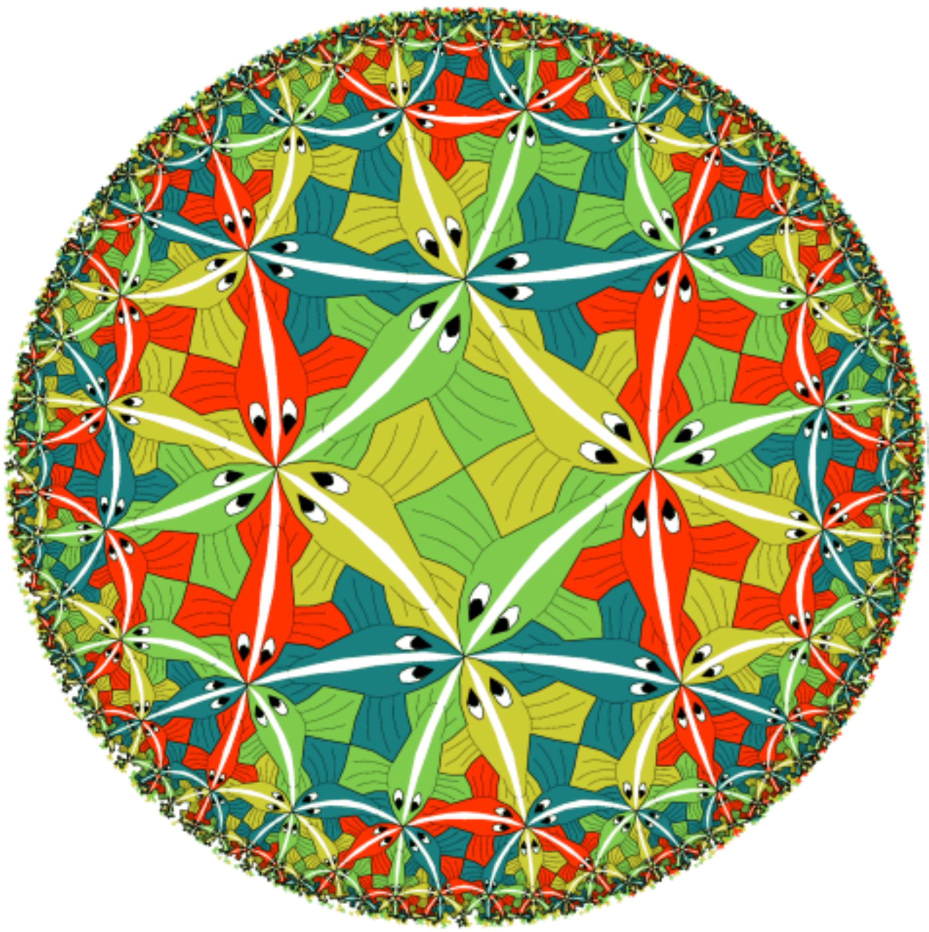


(The gravitational potential does not have a minimum → can have massless excitations
Scale invariant theory → no scale to set the mass)

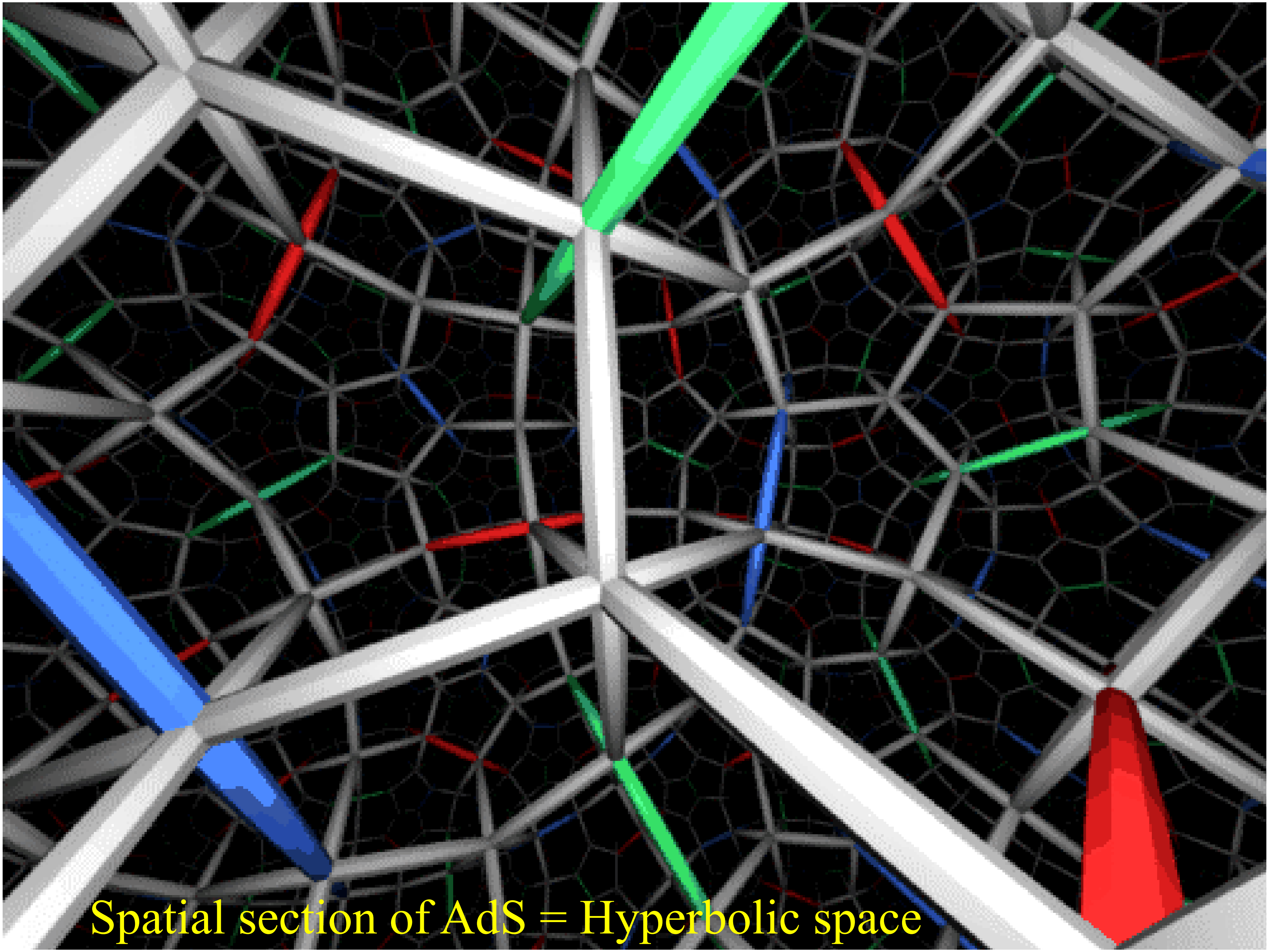
Anti de Sitter space

Solution of Einstein's equations with negative cosmological constant.

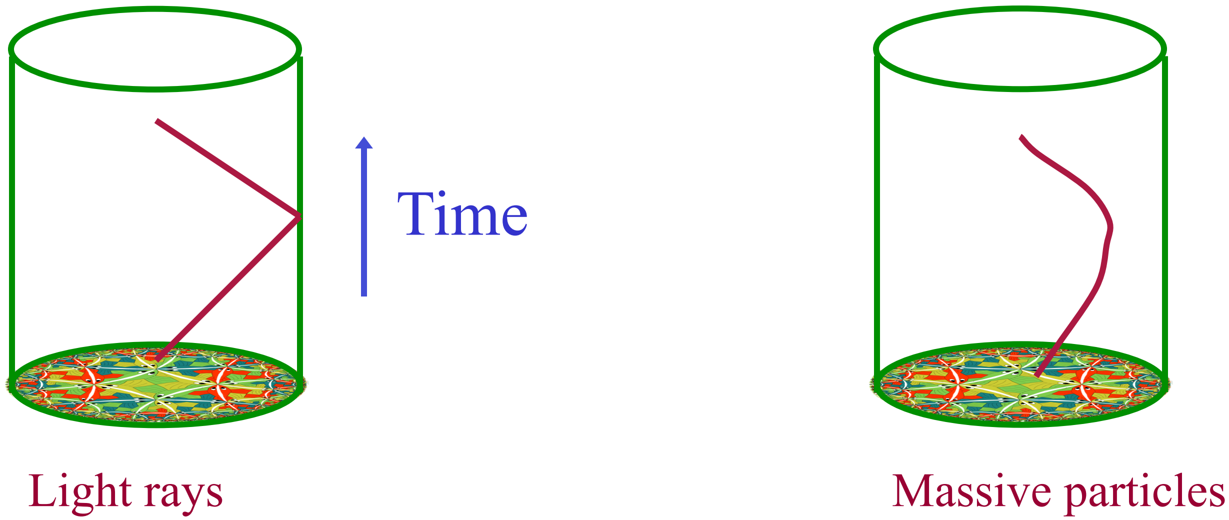
(De Sitter → solution with positive cosmological constant, accelerated expanding universe)



Two dimensional negatively curved space


Spatial section of AdS = Hyperbolic space

R = radius of curvature



The space has a boundary.

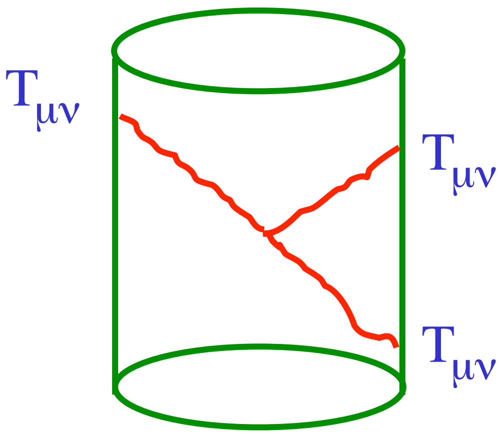
It is infinitely far in spatial distance

A light ray can go to the boundary and back in finite time, as seen from an observer in the interior. The time it takes is proportional to R. (R = curvature radius)

The Field theory is defined on the boundary of AdS.

Building up the Dictionary

Graviton ⟶ stress tensor



Gubser, Klebanov, Polyakov - Witten

We have gravity in the bulk because we have a local stress tensor operator on the boundary theory.

$< T_{\mu\nu}(x)\ T_{\mu\nu}(y)\ \ T_{\mu\nu}(z) >_{\text{Field theory}}$ = Probability amplitude that gravitons go between given points on the boundary

Other operators

⟶ Other fields (particles) propagating in AdS.

Mass of the particle ⟶ scaling dimension of the operator

$$\Delta = 2 + \sqrt{4 + (mR)^2}$$

Most supersymmetric QCD

We expected to have string theory on AdS.

Supersymmetry ⟶ D=10 superstring theory on AdS^5 x $(\text{something})^5$

↓

S^5

Type IIB superstrings on $AdS^5 \times S^5$

(J. Schwarz)

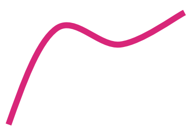
5-form field strength F = generalized magnetic field → quantized

$$\int_{S^5} F = N$$

String Theory

Veneziano
Scherk
Schwarz
Green
…..

Free strings



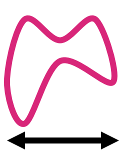
String

$$\text{Tension} = T = \frac{1}{l_s^2} \quad , \qquad l_s = \text{string length}$$

Relativistic, so T = (mass)/(unit length)

Excitations along a stretched string travel at the speed of light

Closed strings



l_s

Can oscillate ⟶ Normal modes ⟶ Quantized energy levels

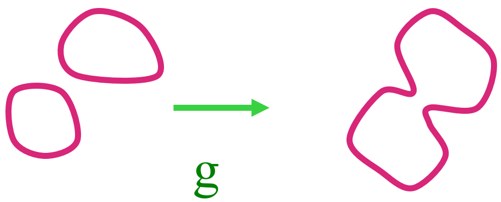
Mass of the object = total energy

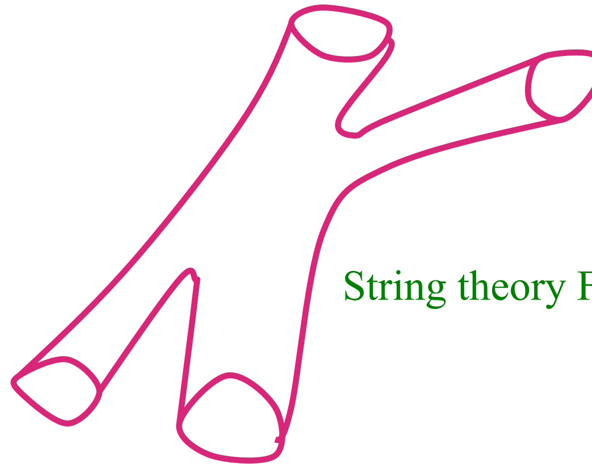
M=0 states include a graviton (a spin 2 particle)

First massive state has $M^2 \sim T$

String Interactions

Splitting and joining





String theory Feynman diagram

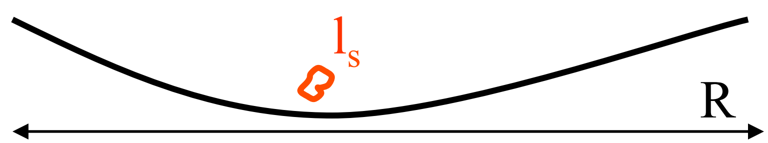
Simplest case: Flat 10 dimensions and supersymmetric

Precise rules, finite results, constrained mathematical structure

At low energies, energies smaller than the mass of the first massive string state

↓

Gravity theory



Radius of curvature >> string length → gravity is a good approximation

(Incorporates gauge interactions → Unification)

Particle theory = gravity theory

Most supersymmetry QCD theory	=	String theory on $AdS_5 \times S^5$

(J.M.)

N colors

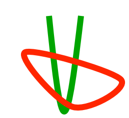
N = magnetic flux through S^5

Radius of curvature

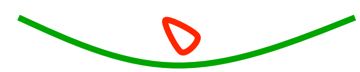
$$R_{S^5} = R_{AdS_5} = \left(g_{YM}^2 \; N\right)^{1/4} l_s$$

Duality:

$g^2 N$ is small → perturbation theory is easy – gravity is bad



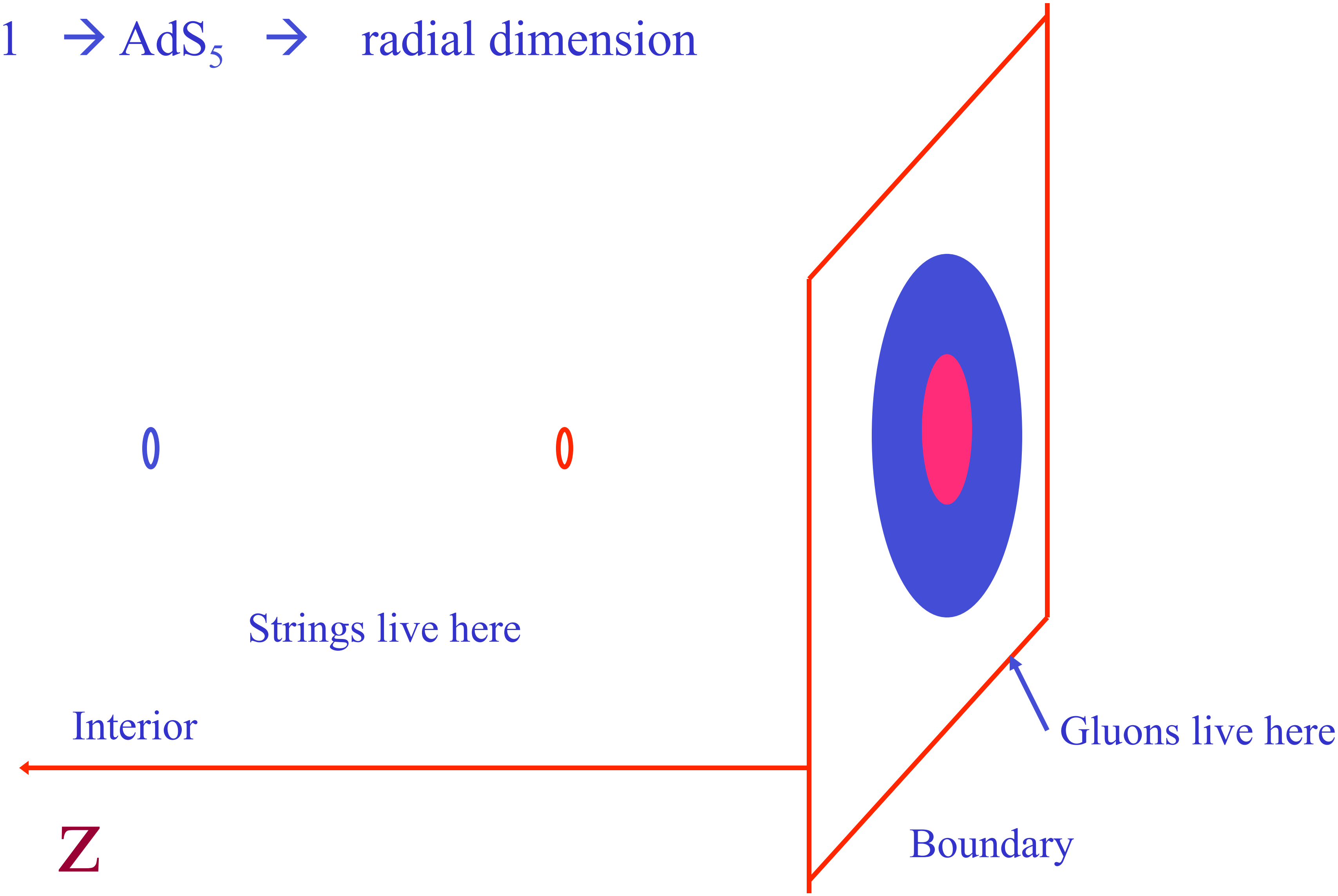
$g^2 N$ is large → gravity is good – perturbation theory is hard



Strings made with gluons become fundamental strings.

Emergent space

3+1 → AdS_5 → radial dimension

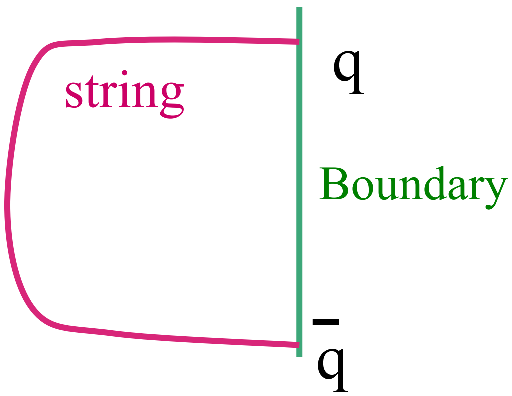


What about the S^5 ?

- Related to the 6 scalars
- $S^5 \rightarrow$ other manifolds = Most susy QCD $\rightarrow$ less susy QCD.
- Large number of examples

Klebanov, Witten,
Gauntlett, Martelli, Sparks,
Hannany, Franco, Benvenutti,
Tachikawa, Yau .....

Quark anti-quark potential



V = potential = proper length of the string in AdS

$$V \approx -\frac{\sqrt{g^2 N}}{L}$$

g^2N À 1

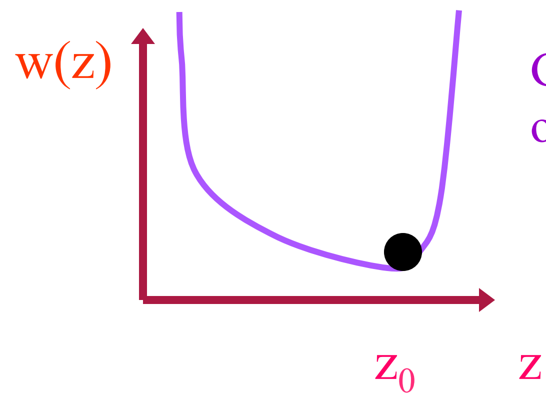
Weak coupling result:

$$V \approx -\frac{g^2 N}{L}$$

g^2N ¿ 1

Confining Theories

Add masses to scalars and fermions → pure Yang Mills at low energies → confining theory. There are many concrete examples.
At strong coupling → gravity solution is a good description.



Gravitational potential or warp factor

Klebanov Strassler

String at z_0 has finite tension from the point of view of the boundary theory.
Graviton in the interior → massive spin=2 particle in the boundary theory = glueball.

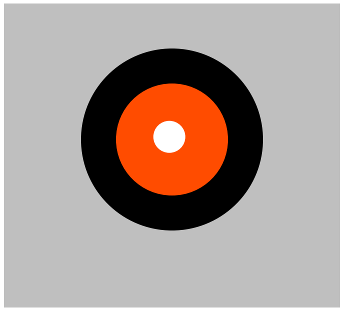
The relation connects a quantum field theory to gravity.

What can we learn about gravity from the field theory ?

- Useful for understanding quantum aspects of black holes

Black holes

Gravitational collapse leads to black holes



Classically nothing can escape once it crosses the event horizon

Quantum mechanics implies that black holes emit thermal radiation. Black holes can be white (Hawking)

$$T \approx \frac{1}{r_s} \approx \frac{1}{G_N M} \qquad T \approx 10^{-8} K \left(\frac{M_{sun}}{M} \right)$$

Black holes evaporate

Evaporation time

$$\tau = \tau_{\text{universe}} \left(\frac{M}{10^{12} Kg} \right)^3$$

Temperature is related to entropy

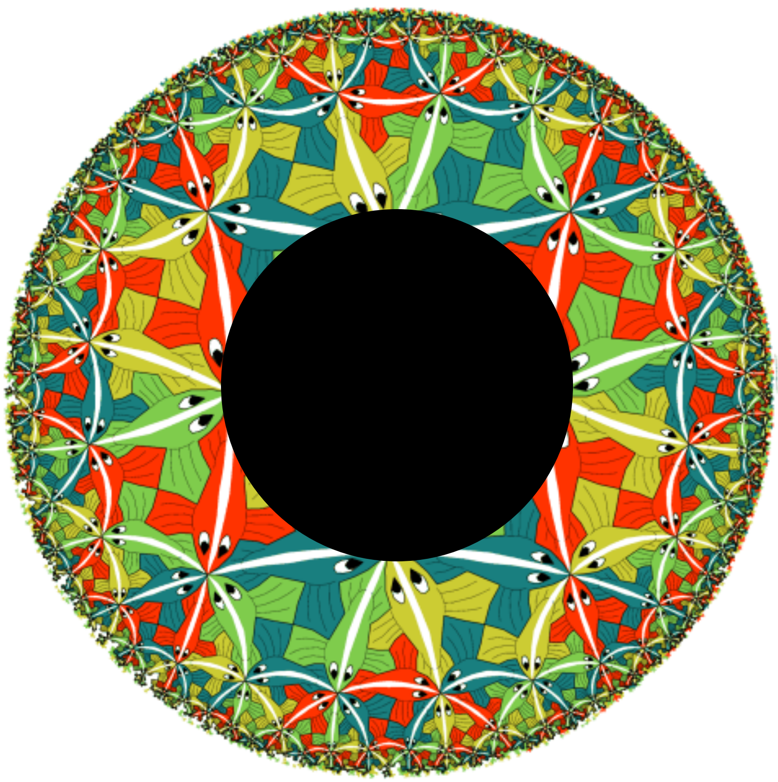
$$dM = T\, dS \qquad S = \frac{\text{Area of the horizon}}{4 L_{\text{Planck}}^2}$$

(Hawking-Bekenstein)

What is the statistical interpretation of this entropy?

Black holes in AdS

Thermal configurations in AdS.



Entropy:

$S_{GRAVITY}$ = Area of the horizon =
$S_{FIELD\ THEORY}$ =
Log[Number of states]

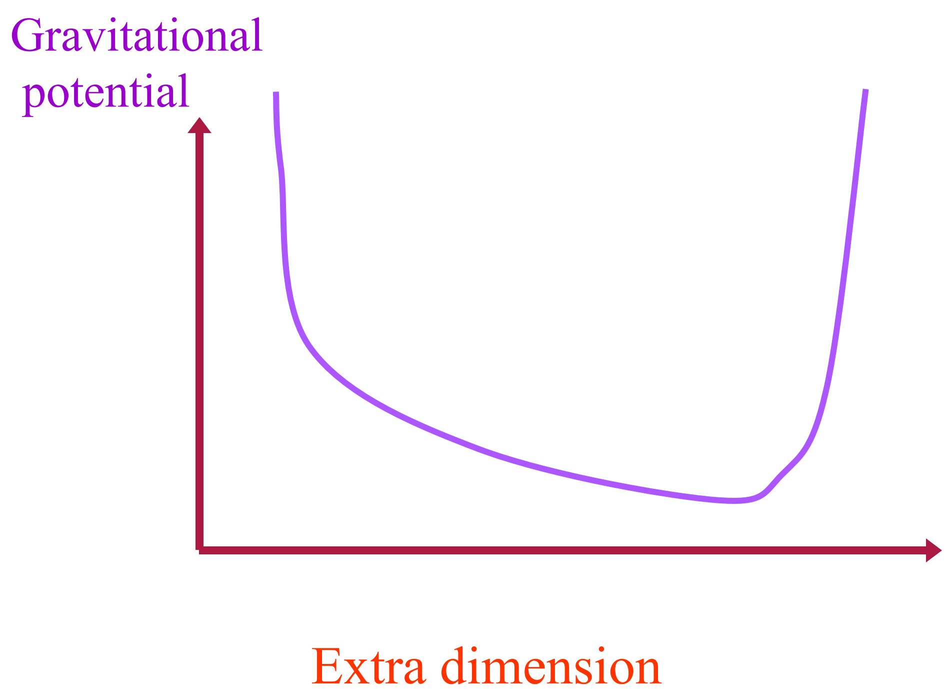
Evolution: Unitary

Solves the information paradox raised by S. Hawking

Confining Theories and Black Holes

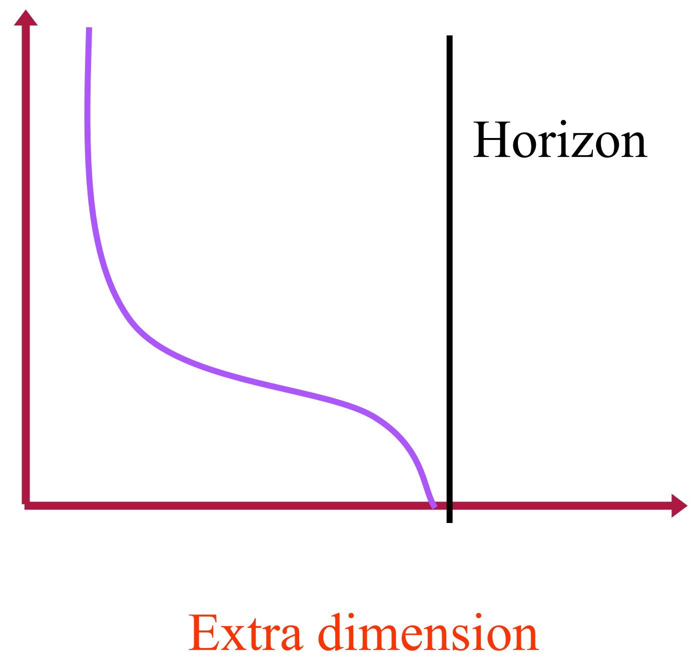
Low temperatures

Confinement

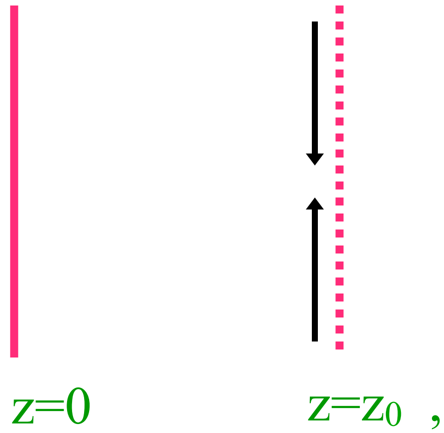


High temperatures

Deconfinement=
black hole (black brane)

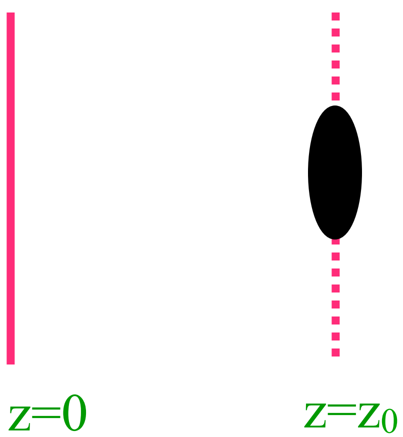


Black holes in the Laboratory ?



QCD → 5d string theory

High energy collision→ produces a black hole =
droplet of deconfined phase ~
quark gluon plasma .



Black hole→ Very low shear viscosity→
similar to what is observed at RHIC:
" the most perfect fluid"

Kovtun, Son, Starinets, Policastro

Very rough model, we do not yet know the precise string theory

A theory of some universe

- Suppose that we lived in anti-de-sitter space
- Then the ultimate description of the universe would be in terms of a 2+1 dimensional field theory living on the sphere at infinity. (With around 10^{120} fields to give a universe of the size of ours)
- Our universe is close to de-Sitter. Could we have a similar description in that case ?

Conclusions

- Gravity and particle physics are "unified"

  Usual: Quantum gravity → particle physics.

  Now: Particle physics ⟷ quantum gravity.

- Black holes and confinement are related
- Emergent space-time. Started from a theory without gravity → got a theory in higher dimensions with gravity.
- Tool to do computations in gauge theories, and source of inspiration for condensed matter physics, or the strong interactions.
- Tool to understand conceptual problems in gravity.

Future

Field theory:

→Theories closer to the theory of strong interactions

→Solve large N QCD.

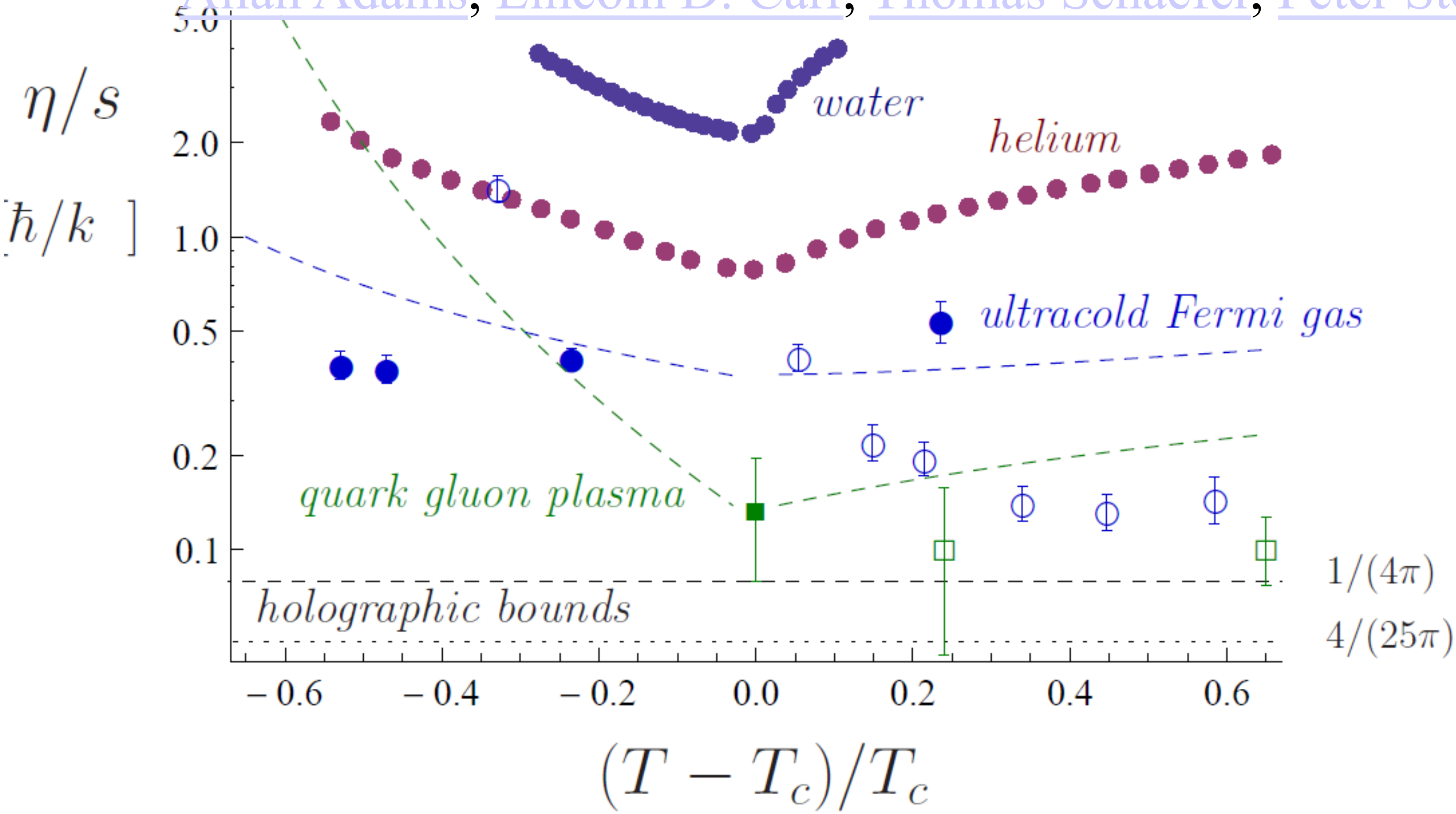
→Apply it to condensed matter physics problems

Gravity:

→Quantum gravity in other spacetimes

→ Understand cosmological singularities

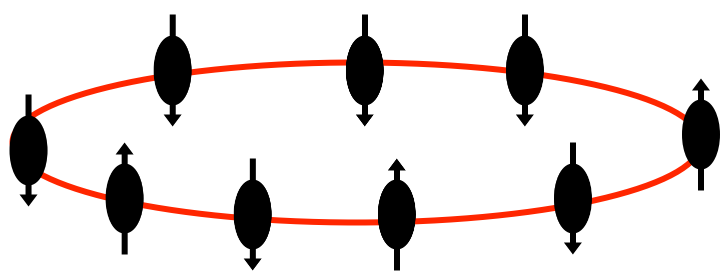
arXiv:1205.5180 [src, pdf, ps, other]
Strongly Correlated Quantum Fluids: Ultracold Quantum Gases, Q
Allan Adams, Lincoln D. Carr, Thomas Schaefer, Peter Steinberg, J



arXiv:1205.5180 [src, pdf, ps, other]
Strongly Correlated Quantum Fluids: Ultracold Quantum Gases, Quantum Chromodyn
Allan Adams, Lincoln D. Carr, Thomas Schaefer, Peter Steinberg, John E. Thomas

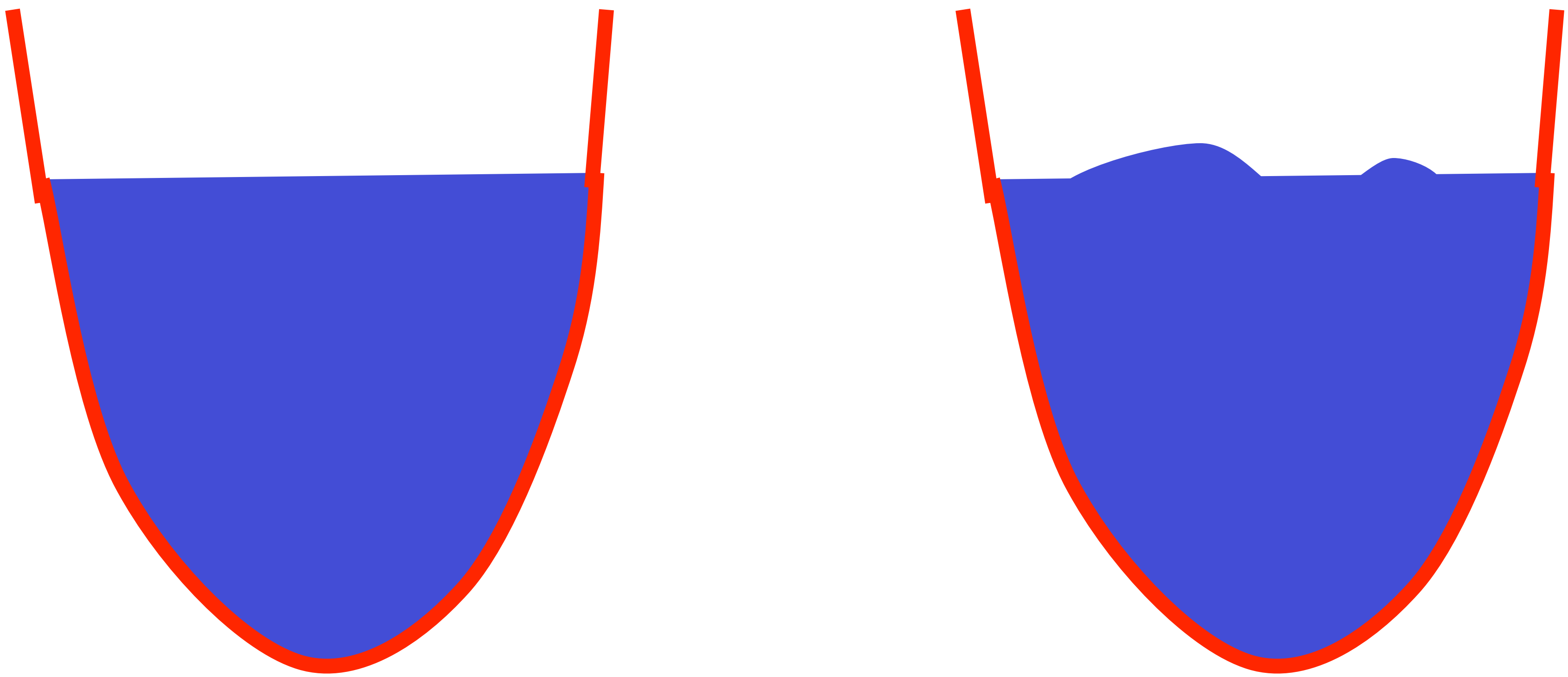
Checking the conjecture

- It is hard because either one side is strongly coupled or the other.
- Supersymmetry allows many checks. Quantities that do not depend on the coupling.
- More recently, ``integrability'' allowed to check the conjecture for quantities that have a non-trivial dependence on the coupling, g^2N.
- One can vividly see how the gluons that live in four dimensions link up to produce strings that move in ten dimensions. …



Minahan, Zarembo, Beisert, Staudacher, Arutyunov, Frolov, Hernandez, Lopez, Eden

Emergent space time



Spacetime: like the fermi surface,
only defined in the classical limit

Lin, Lunin, J.M.