Few-nucleon systems: a lab. for the nuclear interaction

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Starting Point

- Non relativistic Quantum Mechanics

$$H\Psi = E\Psi$$

$$H = T + V$$

$$V = \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k)$$

- Solution of the Schrödinger eq. (bound states)
  - Faddeev equations for $A = 3$
  - Faddeev-Yakubovsky equations for $A = 4$
  - Green Function Monte Carlo $A \leq 12$
  - No Core Shell Model $A \leq 12$
  - Hyperspherical Harmonics

Recent developments in the NN potential

- Nijm I, II, 93 V.G.J. Stocks *et al.*, PRC 49, 2950 (1994)
- AV18 R.B. Wiringa *et al.*, PRC 51, 38 (1995)
- CD Bonn 2000 R. Machleidt, PRC 63, 024001 (2001)

- N^3LO Entem and Machleidt, PRC 68, 041001 (2003)
- N^3LO Epelbaum *et al.*, NPA 747, 362 (2005)

- Low momentum NN interaction $V_{low\ k}$

  S.K. Bogner *et al.*, Phys.Rep. 386, 1 (2003)

  S. Fujii *et al.*, PRC 70, 024003 (2004)

The NN Potential

$$v(NN) = v^{EM}(NN) + v^{\pi}(NN) + v^{R}(NN)$$

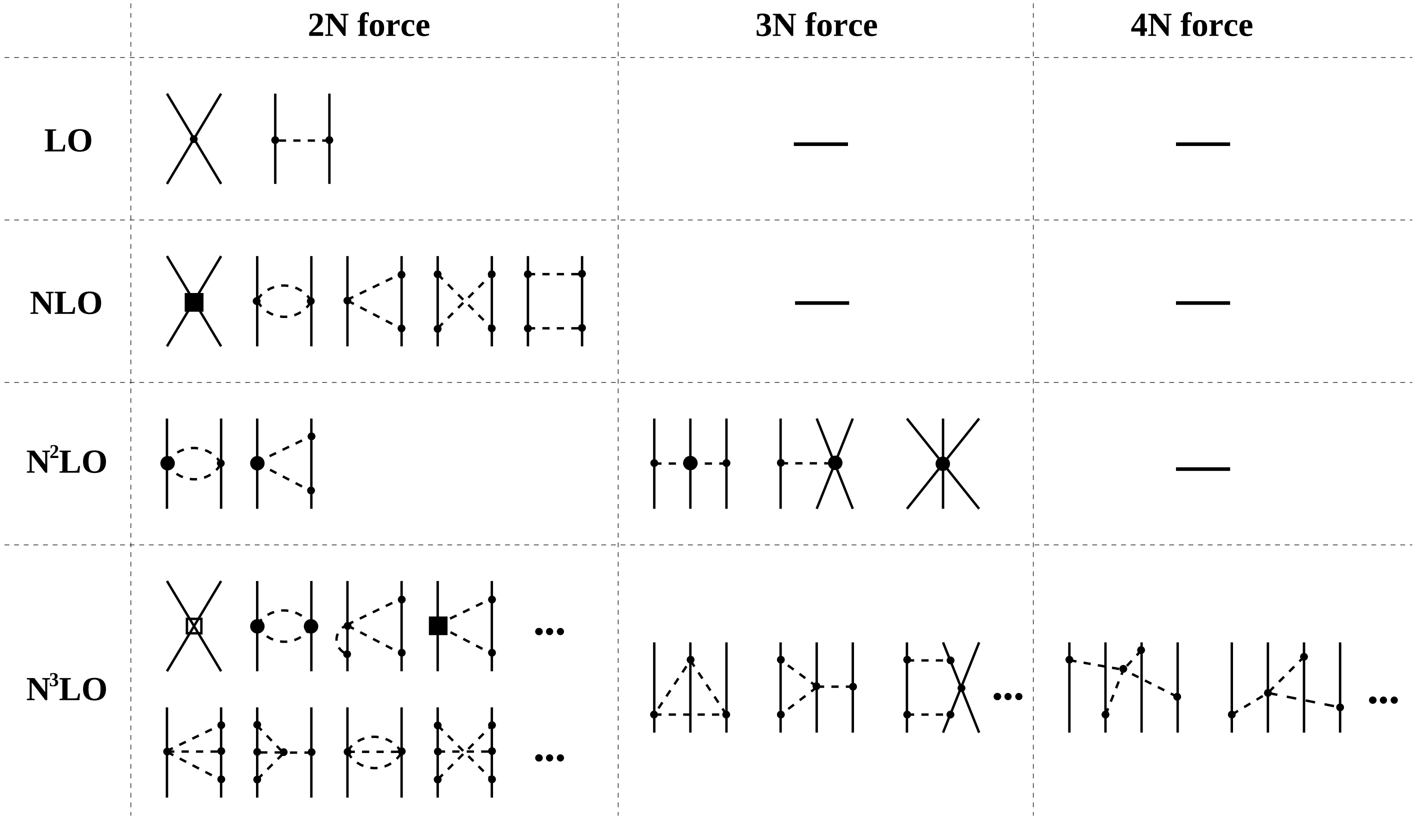
V^{EM} is the electromagnetic part
V^{π} is the one-pion exchange potential
V^{R} is the shor range part and have a certain number of parameters (from 30 to 40) determined by fitting the NN data.

For example the AV18 potential is:

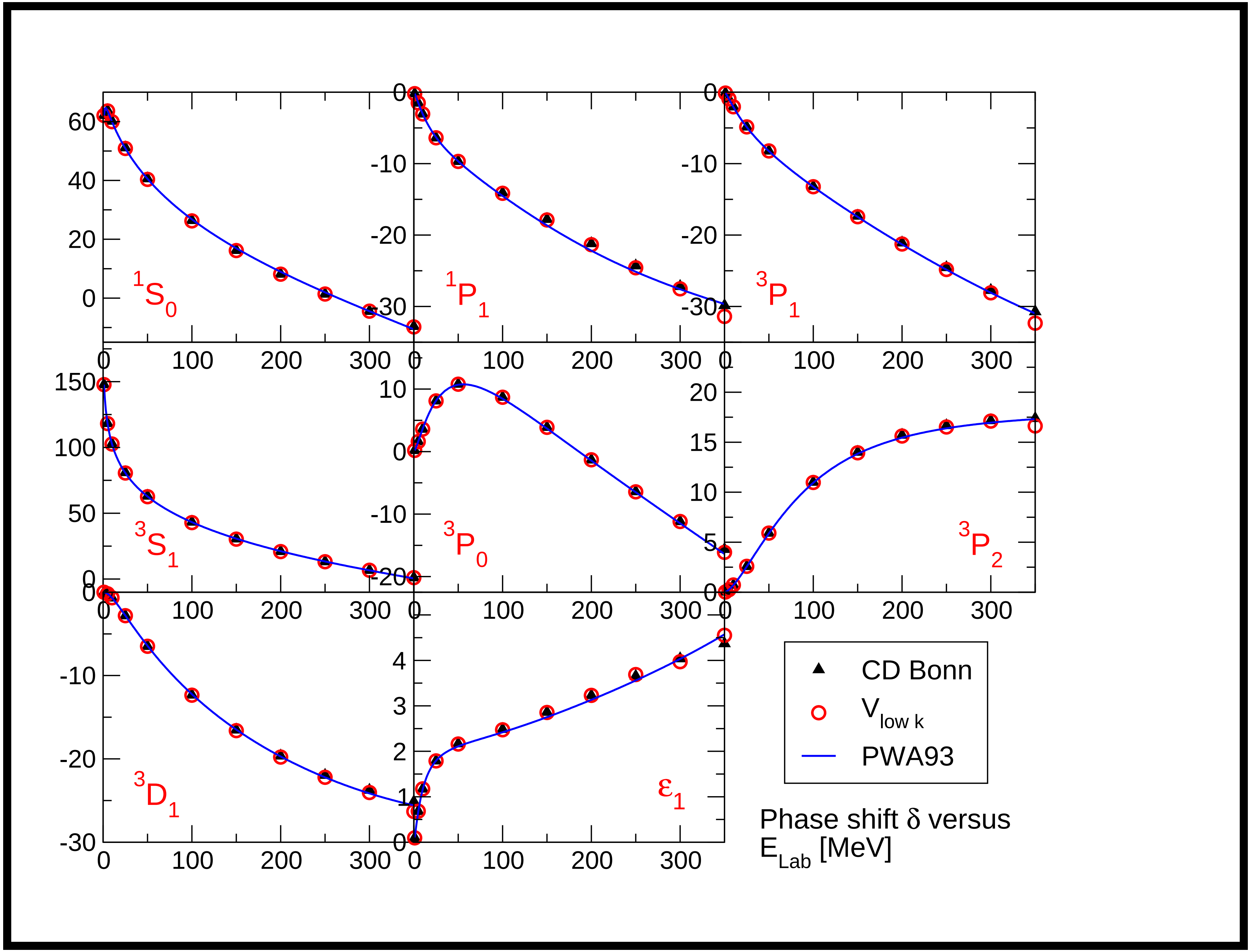
$$v(NN) = \sum_{p=1,18} v_p(r) O^p \quad \text{with}$$

$$O^p = (1, \sigma_1 \cdot \sigma_2, S_{12}, L \cdot S, L^2, L^2 \sigma_1 \cdot \sigma_2, (L \cdot S)^2) \otimes (1, \tau_1 \cdot \tau_2)$$

NN potentials from CHPT



$$V^{\pi}(1,2) = f^2[Y_\mu(r)\sigma_1 \cdot \sigma_2 + T_\mu(r)S_{12}]$$



1S0
1P1
3P1
3S1
3P0
3P2
3D1
ε1
CD Bonn
V low k
PWA93
Phase shift δ versus
E Lab [MeV]

E(MeV)	data	N^3LO	NNLO	NLO	AV18	CD Bonn
0-290	2402 (np)	1.10	10.1	36.2	1.04	1.03
0-290	2057 (pp)	1.50	35.4	80.1	1.30	1.10

Motivation

- Realistic NN potentials describe 2N data with $\chi^2 \approx 1$
- Realistic NN potentials describe 3N data with $\chi^2 >> 1$
- Realistic NN+3N potentials describe 3N data with $\chi^2 >> 1$
- Realistic NN+3N potentials describe 4N data with $\chi^2 >> 1$
- The main disagreements are found in polarization observables even at low energies

Recent developments in the 3N potential

Two-pion Exchange Potentials

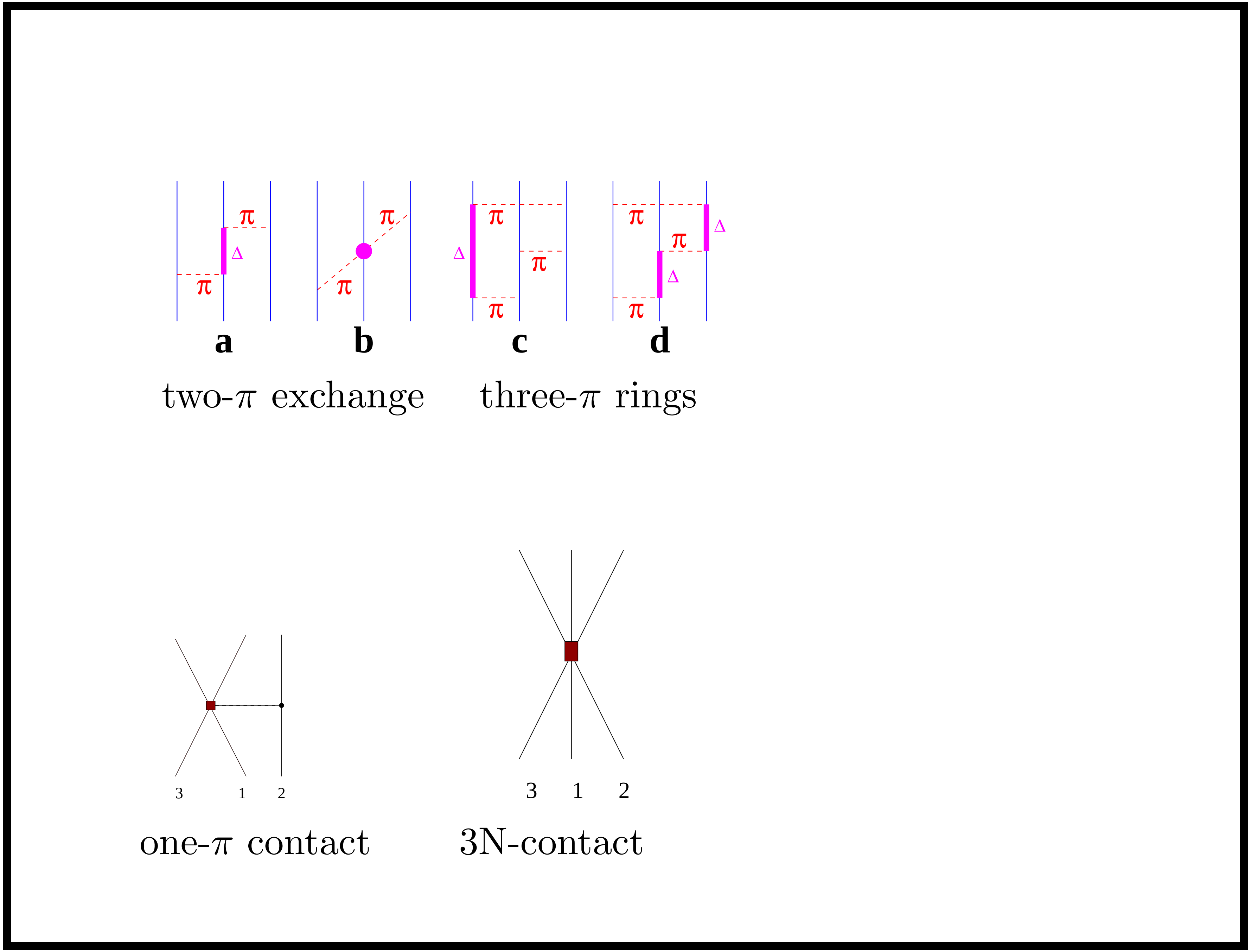
- TM' : S. Coon and Glöckle, PRC 23, 1790 (1981)
- Brazil : H.T. Coelho, T.K.Das and M.R. Robilotta, PRC 28, 1812 (1983)
- URIX : B.S. Pudliner *et al.*, PRL 51, 4396 (1995)

Two-pion Exchange Potentials+rings

- Illinois : S.C. Pieper, PRC 64, 014001 (2001)

3N Potential from chiral EFT

- N2LO non local : E. Epelbaum *et al.*, PRC 66, 064001 (2002)
- N2LO local : P. Navratil, FBS 41, 117 (2007)


π
Δ
π
a
b
c
d
two-π exchange
three-π rings
3
1
2
one-π contact
3N-contact

The 3N Potential

$$W_{3N} = \sum_{i,j,k} W(i,j,k)$$

$$\begin{aligned}
W(1,2,3) = {} & E_1 \tau_1 \cdot \tau_2 (\sigma_1 \cdot r_{31})(\sigma_2 \cdot r_{23}) y(r_{31}) y(r_{23}) \\
& + E_3 \{X_{23}, X_{31}\}\{\tau_2 \cdot \tau_3, \tau_3 \cdot \tau_1\} \\
& + E_4 \left[X_{23}, X_{31}\right] \left[\tau_2 \cdot \tau_3, \tau_3 \cdot \tau_1\right] \\
& + E\ \tau_1 \cdot \tau_1 Z_0(r_{23}) Z_0(r_{31}) \\
& + D\ \tau_1 \cdot \tau_1 \{\sigma_1 \cdot \sigma_2 [y(r_{31}) Z_0(r_{23}) + y(r_{23}) Z_0(r_{31})] \\
& + (\sigma_1 \cdot r_{31})(\sigma_2 \cdot r_{31}) t(r_{31}) Z_0(r_{23}) + (\sigma_1 \cdot r_{23})(\sigma_2 \cdot r_{23}) t(r_{23}) Z_0(r_{31})\}
\end{aligned}$$

$$X_{ij} = t(r_{ij})(\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij}) + y(r_{ij}) \sigma_i \cdot \sigma_j$$

$E, D \neq 0$ from chiral EFT

Experimental and GFMC energies

	method	GFMC	GFMC	GFMC	NCSM
nucleus	Exp.	AV18	AV18+UR	AV18+IL4	N3LO+N2LO
^{2}H(1^+)	-2.2245	-2.2245			
^{3}H$(\frac{1}{2}^+)$	-8.48	-7.61	-8.46	-8.44	-8.47
^{3}He$(\frac{1}{2}^+)$	-7.72	-6.87	-7.71	-7.69	-7.73
^{4}He(0^+)	-28.30	-24.07	-28.33	-28.35	-28.36
^{6}Li(1^+)	-31.99	-26.9	-31.1	-32.0	-32.63
^{7}Li$(\frac{3}{2}^-)$	-39.24	-31.6	-37.5	-39.5	
^{7}Li$(\frac{1}{2}^-)$	-38.77	-31.1	-32.1	-39.0	

Topics

- Construction of $A = 3, 4$ bound states
- Construction of $A = 3, 4$ scattering states
- Production of benchmarks:
  a) bound states, b) scattering states
- Calculations of observables
- Theory vs experiment
- $n - d$, $p - d$, $N - {}^3\mathrm{H}$, $N - {}^3\mathrm{He}$
- capture reactions as $p + d \rightarrow {}^3\mathrm{He} + \gamma$ or $n + d \rightarrow {}^3\mathrm{H} + \gamma$

$A = 3, 4$ bound states

The A=3,4 wave function is written as

$$\Psi_3 = \sum_{i=1,3} \psi(\mathbf{x}_i, \mathbf{y}_i)$$

$$\Psi_4 = \sum_{i=1,12} \psi(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)$$

$$\psi(\mathbf{x}_i, \mathbf{y}_i) = \sum_{\alpha=1}^{N_c} \phi_\alpha(x_i, y_i) \mathcal{Y}_\alpha(i, j, k)$$

$$\psi(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i) = \sum_{\alpha=1}^{N_c} \phi_\alpha(x_i, y_i, z_i) \mathcal{Y}_\alpha(i, j, k, m)$$

Jacobi Coordinates:

$A = 3$

$$\mathbf{x}_i = \mathbf{r}_j - \mathbf{r}_k$$

$$\mathbf{y}_i = \sqrt{\tfrac{4}{3}}\left(\tfrac{\mathbf{r}_j+\mathbf{r}_k}{2} - \mathbf{r}_i\right)$$

$A = 4$

$$\mathbf{x}_i = \mathbf{r}_j - \mathbf{r}_k$$

$$\mathbf{y}_i = \sqrt{\tfrac{4}{3}}\left(\tfrac{\mathbf{r}_j+\mathbf{r}_k}{2} - \mathbf{r}_i\right)$$

$$\mathbf{z}_i = \sqrt{\tfrac{3}{2}}\left(\tfrac{\mathbf{r}_j+\mathbf{r}_k+\mathbf{r}_i}{3} - \mathbf{r}_m\right)$$

Hyperspherical Variables:

$$x_i = \rho\cos\theta_i$$

$$y_i = \rho\sin\theta_i$$

$$[\rho,\Omega_i] = [\rho,\theta_i,\hat{x}_i,\hat{y}_i]$$

$$x_i = \rho\cos\theta_{1i}$$

$$y_i = \rho\sin\theta_{1i}\cos\theta_{2i}$$

$$z_i = \rho\sin\theta_{1i}\sin\theta_{2i}$$

$$[\rho,\Omega_i] = [\rho,\theta_{1i},\theta_{2i},\hat{x}_i,\hat{y}_i,\hat{z}_i]$$

$$T = -\sum_{i=1}^{A} \frac{\hbar^2}{2m_i}\nabla_i^2 = -\frac{\hbar^2}{m}\sum_{i=1}^{N}\nabla_{x_i}^2 - \frac{\hbar^2}{2M}\nabla_{\mathbf{X}}^2$$

$$\sum_{i=1}^{N}\nabla_i^2 = \left(\frac{\partial^2}{\partial\rho^2} + \frac{3N-1}{\rho}\frac{\partial}{\partial\rho} + \frac{L_N^2(\Omega_N)}{\rho^2}\right)$$

The HH functions are:

$$L_3^2(\Omega)\mathcal{Y}_{[K]} = K(K+4)\mathcal{Y}_{[K]}$$

$$\mathcal{Y}_{[K]} = \mathcal{N}_{l_1 l_2}^{n}\ {}^{(2)}P_n^{l_1 l_2}(\cos 2\phi)[Y_{l_1}(x_1)Y_{l_2}(x_2)]_{LM}$$

$$L_4^2(\Omega)\mathcal{Y}_{[K]} = K(K+7)\mathcal{Y}_{[K]}$$

$$\mathcal{Y}_{[K]} = \mathcal{N}_{l_1 l_2}^{n_1 n_2}\ {}^{(3)}P_{n_1 n_2}^{l_1 l_2 l_3}(\cos 2\phi)[Y_{l_1}(x_1)Y_{l_2}(x_2)Y_{l_3}(x_3)]_{LM}$$

HH basis

The $A = 3$ amplitudes are expanded in the HH basis

$$\phi_\alpha(x_i, y_i) = \mathrm{e}^{-\beta\rho}\left[\sum_{mn} A^{\alpha}_{mn}\mathcal{L}^{(5)}_m(\rho)\ {}^{(2)}P^{l_{1\alpha},l_{2\alpha}}_n(\theta_i)\right]$$

$$\phi_\alpha(x_i, y_i, z_i) = \mathrm{e}^{-\beta\rho}\left[\sum_{mn_1n_2} A^{\alpha}_{mn_1n_2}\mathcal{L}^{(8)}_m(\rho)\ {}^{(3)}P^{l_{1\alpha},l_{2\alpha},l_{3\alpha}}_{n_1n_2}(\theta_{1i}, \theta_{2i})\right]$$

The A=3,4 HH basis elements are

$$|\alpha m[K] >= \mathcal{L}^{(5)}_m(\rho)\mathrm{e}^{-\beta\rho}\sum_i {}^{(2)}P^{l_{1\alpha},l_{2\alpha}}_n(\theta_i)\mathcal{Y}_\alpha(i, j, k)$$

$$|\alpha m[K] >= \mathcal{L}^{(8)}_m(\rho)\mathrm{e}^{-\beta\rho}\sum_i {}^{(3)}P^{l_{1\alpha},l_{2\alpha},l_{3\alpha}}_{n_1n_2}(\theta_{1i}, \theta_{2i})\mathcal{Y}_\alpha(i, j, k, m)$$

$K = l_{1\alpha} + l_{2\alpha} + 2n$ $\qquad$ $K = l_{1\alpha} + l_{2\alpha} + l_{3\alpha} + 2n_1 + 2n_2$

$A = 3, 4$ basis elements

The A=3,4 bound state wave functions result

$$\Psi_{A=3,4} = \sum_{\alpha,m,[K]} A^{\alpha}_{m[K]} |\alpha\, m\, [K] >$$

$|\alpha\, m\, [K] >$ is a complete antisymmetric basis element.

The coefficients $A^{\alpha}_{m[K]}$ are obtained from the generalized eigenvalue problem:

$$\sum_{\alpha,m,[K]} A^{\alpha}_{m[K]} < \alpha'\, m'\, [K'] | H - E | \alpha\, m\, [K] >= 0$$

The HH basis can be defined in configuration or momentum space:

$$< \rho, \Omega | \alpha \, m \, [K] >= \mathcal{L}_m^{(A)}(\rho) \; \mathrm{e}^{-\beta\rho} B_{[K]}^{\alpha}(\Omega)$$

$$< Q, \Omega_p | \alpha \, m \, [K] >= \mathcal{F}_{K m}^{(A)}(Q) B_{[K]}^{\alpha}(\Omega_q)$$

with

$$\mathcal{F}_{KM}^{(A)}(Q) = (-i)^K \int_0^\infty d\rho \frac{\rho^{3N-1}}{(Q\rho)^{3N/2-1}} J_{K+3N/2-1}(Q\rho) \mathcal{L}^{(A)}(\rho)$$

The T-matrix elements $< \alpha' \, m' \, [K'] | T | \alpha \, m \, [K] >$ are analytical

The V-matrix elements $< \alpha' \, m' \, [K'] | V | \alpha \, m \, [K] >$ can be calculated in configuration or momentum space as convenience.

Potential	Method	$B(^3\mathrm{H})$	$B(^4\mathrm{He})$
AV18	HH	7.624	24.22
	FE/FY Bochum	7.621	24.23
	FE/FY Lisbon	7.621	24.24
CDBonn	HH	7.998	26.13
	FE/FY Lisbon	7.998	26.11
	NCSM	7.99(1)	
N3LO-Idaho	HH	7.854	25.38
	FE/FY Lisbon	7.854	25.38
	NCSM	7.852(5)	25.39(1)
AV18/UIX	HH	8.479	28.47
	FE/FY Bochum	8.476	28.53
	GFMC	8.46(1)	28.33(2)
CDBonn/TM	HH	8.474	29.00
	FE/FY Bochum	8.482	29.09
N3LO-Idaho/N2LO	HH	8.474	28.37
	NCSM	8.473(5)	28.34(2)

^{3}H	B	$\langle T\rangle$	r_p	r_n	$P_{S'}$	P_P	P_D	$P_{T3/2}$
AV18	7.624	46.727	1.653	1.824	1.293	0.066	8.510	0.0025
CDBonn	7.998	37.630	1.618	1.771	1.310	0.047	7.018	0.0049
N3LO-I	7.854	34.555	1.655	1.808	1.365	0.037	6.312	0.0009
AV18/UIX	8.479	51.275	1.582	1.732	1.054	0.135	9.301	0.0025
CDBonn/TM	8.474	39.364	1.580	1.722	1.202	0.101	6.971	0.0049
N3LO-I/N2LO	8.474	36.482	1.611	1.752	1.242	0.121	6.815	0.0009
Exp.	8.482		1.60					

^{3}He	B	$\langle T\rangle$	r_p	r_n	$P_{S'}$	P_P	P_D	$P_{T3/2}$
AV18/UIX	7.750	50.211	1.771	1.602	1.242	0.132	9.248	0.0075
CDBonn/TM	7.720	38.495	1.767	1.597	1.409	0.099	6.966	0.0106
N3LO-I/N2LO	7.733	35.745	1.794	1.628	1.450	0.119	6.818	0.0057
Exp.	7.718		1.77					

^{4}He	B	$\langle T\rangle$	r_p	r_n	P_P	P_D	P_{T1}	P_{T2}
AV18/UIX	28.46	113.30	1.430	1.425	0.732	16.03	0.0025	0.0050
CDBonn/TM	29.00	84.56	1.396	1.391	0.454	9.94	0.0021	0.0105
N3LO-I/N2LO	28.36	74.93	1.476	1.471	0.608	10.79	0.0028	0.0020
Exp.	28.30		1.47					

$A = 3, 4$ scattering states

The A=3,4 scattering wave function is written as

$$\Psi = \Psi_C + \Psi_A$$

The internal part is

$$\Psi_C^{A=3} = \sum_{i=1,3} \psi(\mathbf{x}_i, \mathbf{y}_i) \; , \qquad \Psi_C^{A=4} = \sum_{i=1,12} \psi(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)$$

The i-amplitude has total JJ_z and can be expanded in the HH basis.

$$\Psi_C(A = 3, 4) = \sum_{\alpha, m, [K]} A^{\alpha}_{m[K]} |\alpha\, m\, [K] >$$

The second term, Ψ_A describes the asymptotic motion of a bound state relative to the incident nucleon.

Asymptotic state

Ψ_A can be written as a sum of i amplitudes with the generic one having the form

$$\Omega^{\lambda}_{LSJ}(\mathbf{x}_i,\mathbf{y}_i) = \mathcal{R}^{\lambda}_L(y_i)\left\{[\phi_d(\mathbf{x}_i)s^i]_S Y_L(\hat{y}_i)\right\}_{JJ_z}[t_d^{jk}t^i]_{TT_z}$$

$$\Omega^{\lambda}_{LSJ}(\mathbf{x}_i,\mathbf{y}_i,\mathbf{z}_i) = \mathcal{R}^{\lambda}_L(z_i)\left\{[\phi_3(\mathbf{x}_i,\mathbf{y}_i)s^m]_S Y_L(\hat{z}_i)\right\}_{JJ_z}[t_3^{ijk}t^m]_{TT_z}$$

The functions $\mathcal{R}^{\lambda}$ are related to the regular or irregular Coulomb (spherical Bessel) functions. The functions Ω^{λ} can be combined to form a general asymptotic state $^{(2S+1)}L_J$

$$\Omega^{+}_{LSJ} = \Omega^{0}_{LSJ} + \sum_{L'S'} {}^J\mathcal{S}^{SS'}_{LL'}\Omega^{1}_{L'S'J}$$

The Kohn Variational Principle

The $A = 3, 4$ scattering w.f. for an incident state with relative angular momentum L, spin S and total angular momentum J has the form

$$\Psi^+_{LSJ} = \sum_{\alpha,m,[K]} A^\alpha_{m[K]} |\alpha\, m\, [K] > + \Omega^0_{LSJ} + \sum_{L'S'} {}^J\mathcal{S}^{SS'}_{LL'} \Omega^1_{L'S'J}$$

A variational estimate of the trial parameters $A^\alpha_{m[K]}$, ${}^J\mathcal{S}^{SS'}_{LL'}$ can be obtained using the Kohn Variational Principle

$$[{}^J\mathcal{S}^{SS'}_{LL'}] = {}^J\mathcal{S}^{SS'}_{LL'} - \mathrm{i}\langle \Psi^-_{LSJ} | H - E | \Psi^+_{L'S'J} \rangle$$

Calculation of Observables

The observables can be calculated from the transition matrix M decomposed as a sum of the Coulomb plus a nuclear term

$$M_{\nu\nu'}^{SS'}(\theta) = f_c(\theta)\delta_{SS'}\delta_{\nu\nu'} + \frac{\sqrt{4\pi}}{k}\sum_{L,L',J}\sqrt{2L+1}(L0S\nu|J\nu)$$

$$\times(L'M'S'\nu'|J\nu)\exp[i(\sigma_L+\sigma_{L'}-2\sigma_0)]\ {}^JT_{LL'}^{SS'}\ Y_{L'M'}(\theta,0)$$

$$\sigma = \frac{tr\{MM^\dagger\}}{6} \qquad \sigma = \frac{tr\{MM^\dagger\}}{4}$$

$$A_y = \frac{tr\{M\sigma_y M^\dagger\}}{6} \qquad A_{y0} = \frac{tr\{M\sigma_y M^\dagger\}}{4}$$

$$iT_{11} = \frac{tr\{MS_y M^\dagger\}}{6} \qquad A_{0y} = \frac{tr\{MS_y M^\dagger\}}{4}$$

The scattering lengths are: ${}^{(2J+1)}a_{NY} = -\lim_{q\to 0}\mathcal{R}_{0J,0J}^J$

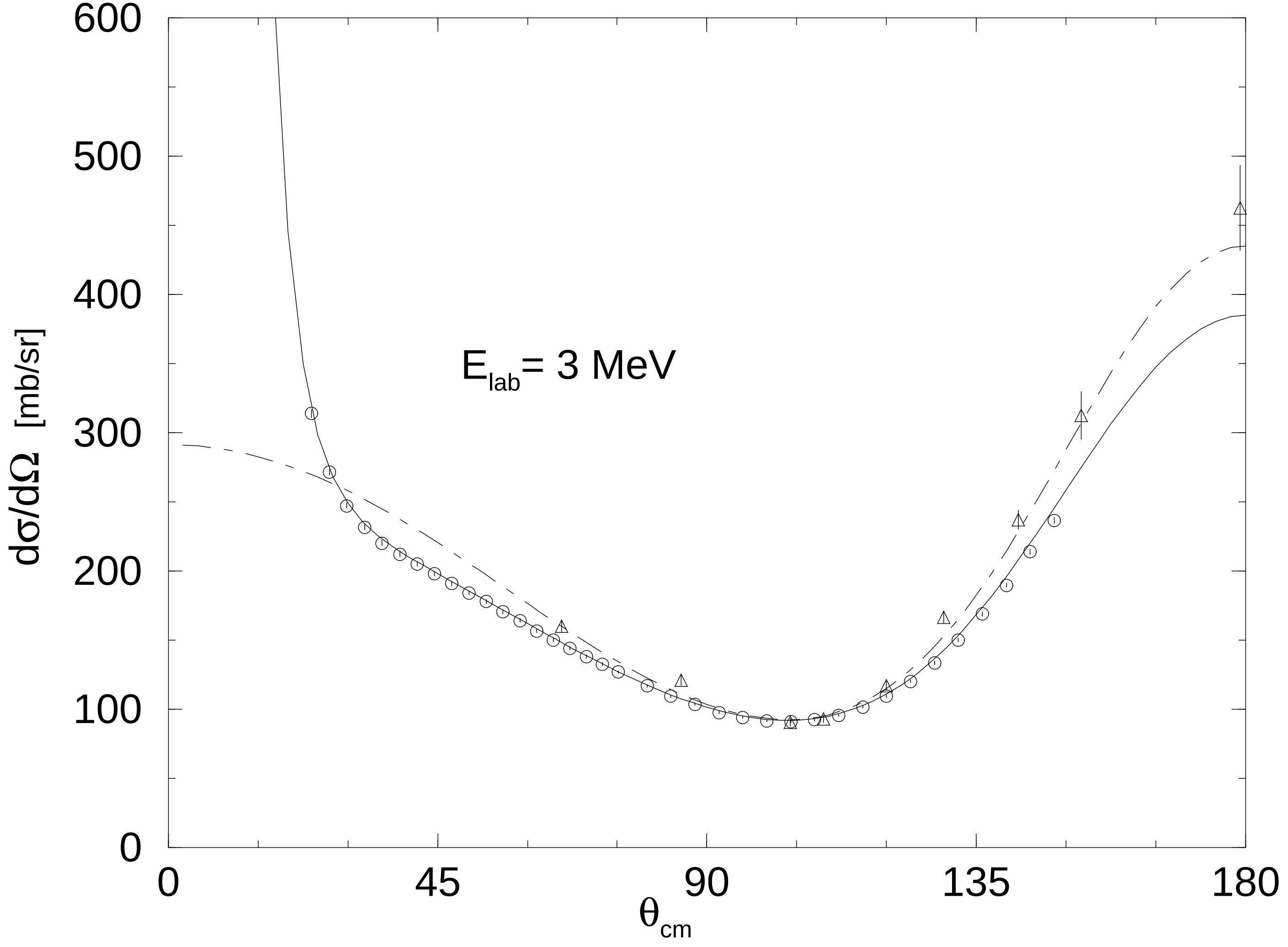
Table 1: The *nd* doublet and quartet scattering lengths ${}^2a_{nd}$, ${}^4a_{nd}$

Potential Model	${}^2a_{nd}$[fm]	${}^4a_{nd}[fm]$
AV14 HH	1.189	6.379
AV14 FE Los Alamos	1.204	6.380
AV18 HH	1.258	6.345
AV18 FE Bochum	1.248	6.346
AV18* PHH	1.275	6.325
AV18* FE Bochum	1.263	6.326
N3LO-Idaho HH	1.100	6.342
AV14/TM HH	0.586	6.371
AV18/UIX HH	0.590	6.343
AV18/UIX FE Bochum	0.578	6.347
AV18*/UIX HH	0.610	6.323
AV18*/UIX FE Bochum	0.597	6.326
N3LO-Idaho/N2LO HH	0.675	6.342
Exp.	0.65±0.04	6.35±0.02
Exp.	0.645±0.003±0.007	

Table 2: The *pd* doublet and quartet scattering lengths ${}^2a_{pd}$, ${}^4a_{pd}$

Potential Model	${}^2a_{pd}$	${}^4a_{pd}$	${}^2a_{pd}(T=1/2)$	${}^4a_{pd}(T=1/2)$
AV14 HH	0.937	13.773	0.941	13.773
AV14 FE Los Alamos			0.965	13.764
AV18 HH	1.134	13.662	1.150	13.662
AV18* HH	1.185	13.588	1.198	13.589
N3LO-Idaho HH	0.876	13.646	0.866	13.646
AV18/UIX PHH	-0.089	13.662	-0.074	13.663
AV18*/UIX PHH	-0.035	13.588	-0.019	13.590
N3LO-Idaho/N2LO HH	0.072	13.647	0.082	13.647

N-d cross section at E=3 MeV



N-d cross section at E=3 MeV

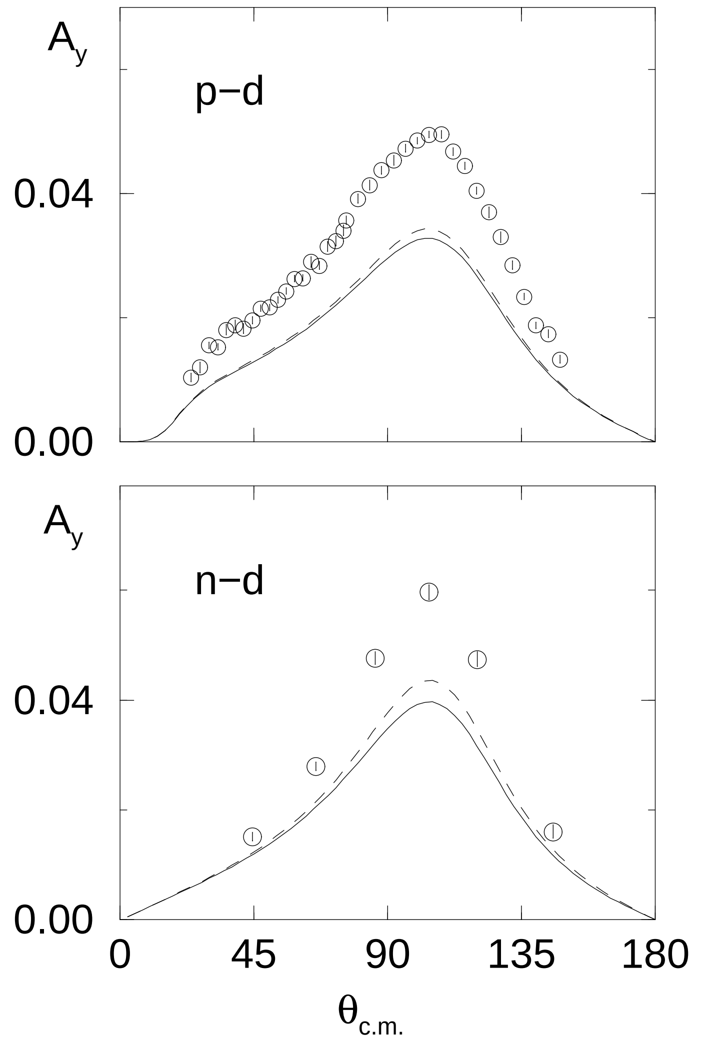
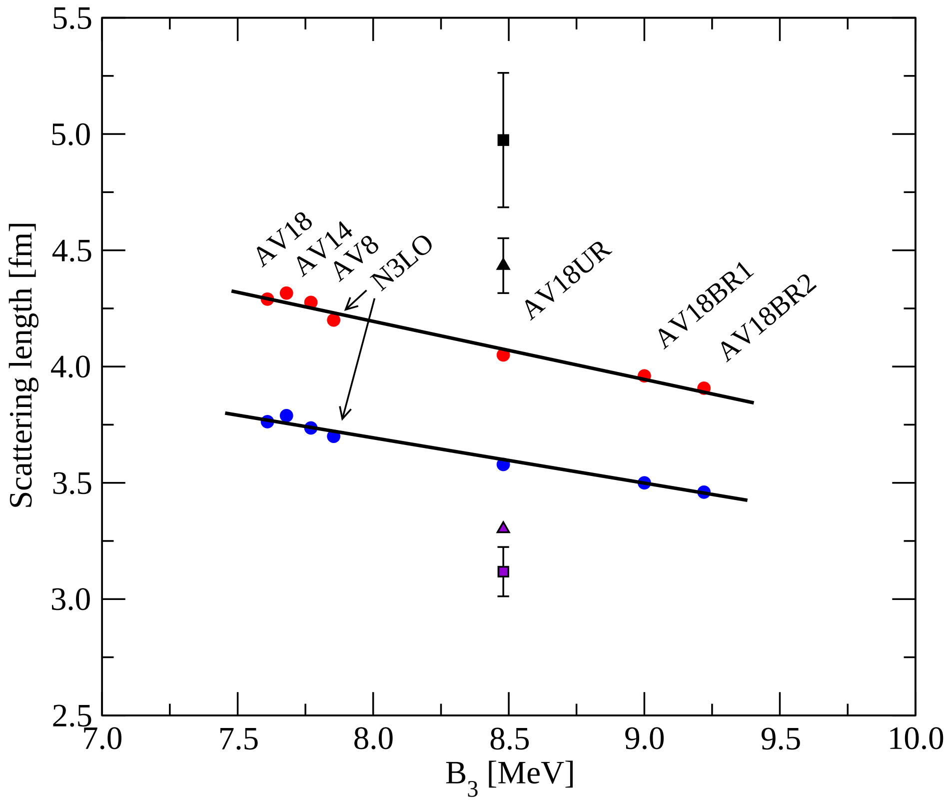


Table 3: The n^3H, p^3He singlet and triplet scattering lengths ${}^1a_{n^3\mathrm{H}}$, ${}^3a_{n^3\mathrm{H}}$, ${}^1a_{p^3\mathrm{He}}$, ${}^3a_{p^3\mathrm{He}}$

Potential Model	${}^1a_{n^3\mathrm{H}}$	${}^3a_{n^3\mathrm{H}}$	${}^1a_{p^3\mathrm{He}}$	${}^3a_{p^3\mathrm{He}}$
AV18 HH	4.29	3.73	12.9	10.0
AV18 FY	4.27	3.71		
AV18 FY	4.28	3.71		
N3LO-I HH	4.20	3.67		
N3LO-I FY	4.23	3.67		
AV18/UIX HH	4.10	3.61	11.5	9.13
AV18/UIX FY	4.04	3.60		
N3LO-I/N2LO HH	3.99	3.54		
Exp.	4.98 ± 0.29	3.13 ± 0.11	10.8 ± 2.6	8.1 ± 0.5
	4.45 ± 0.10	3.32 ± 0.02		10.2 ± 1.5

Zero Energy $n -^3\mathrm{H}$ scattering length



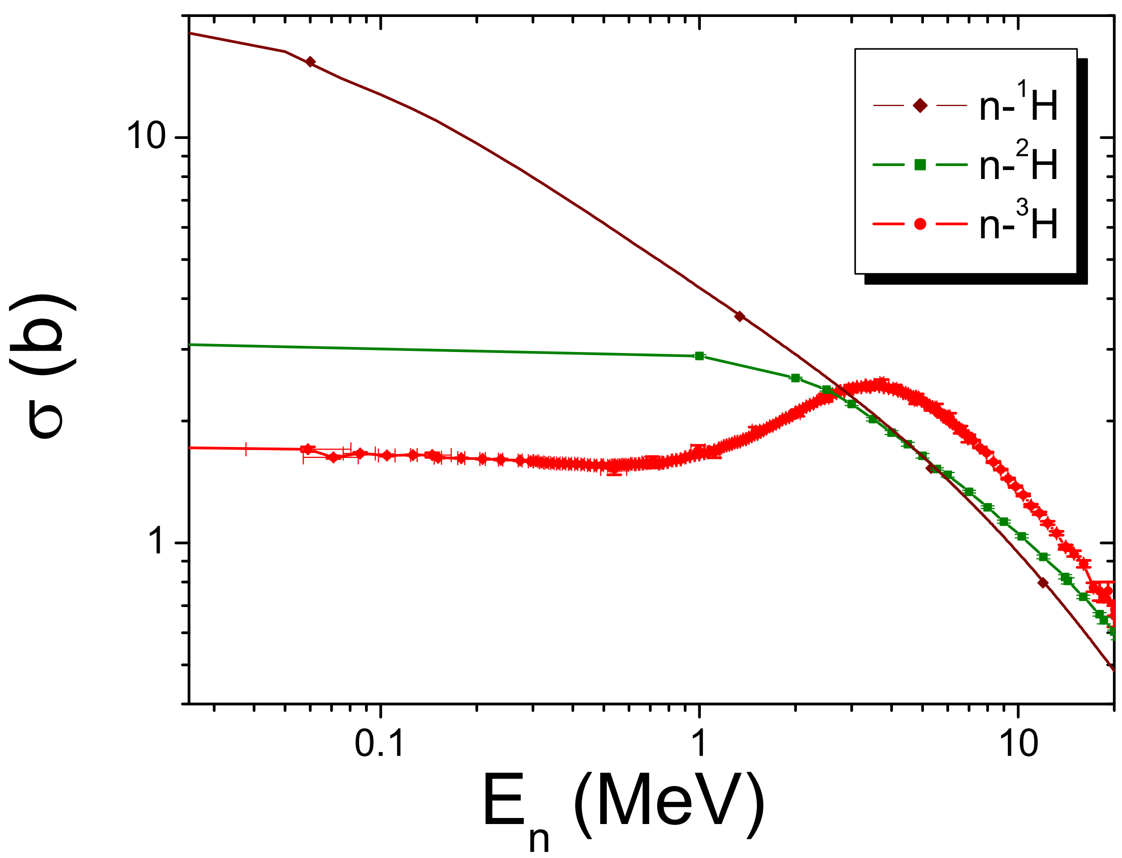
The experimental quantities are:

$$\sigma_T = \pi(|a_s|^2 + 3|a_t|^2),$$

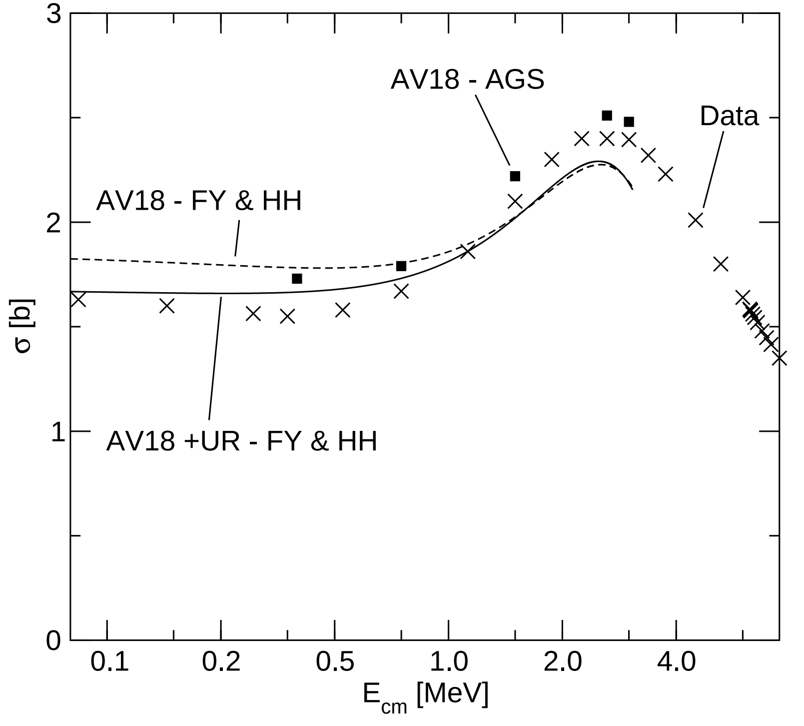
$$a_c = \frac{1}{4}a_s + \frac{3}{4}a_t$$

Model	σ_T(b)	a_c(fm)	a_s(fm)	a_t(fm)
AV14+UR	1.74	3.71	4.10	3.58
AV18+UR	1.73	3.71	4.08	3.58
Expt.	1.70 ± 0.03	3.82 ± 0.07		
		3.59 ± 0.02	4.97 ± 0.29	3.12 ± 0.11
		3.607 ± 0.02	4.43 ± 0.12	3.30 ± 0.01

$n -^3$ H total cross section



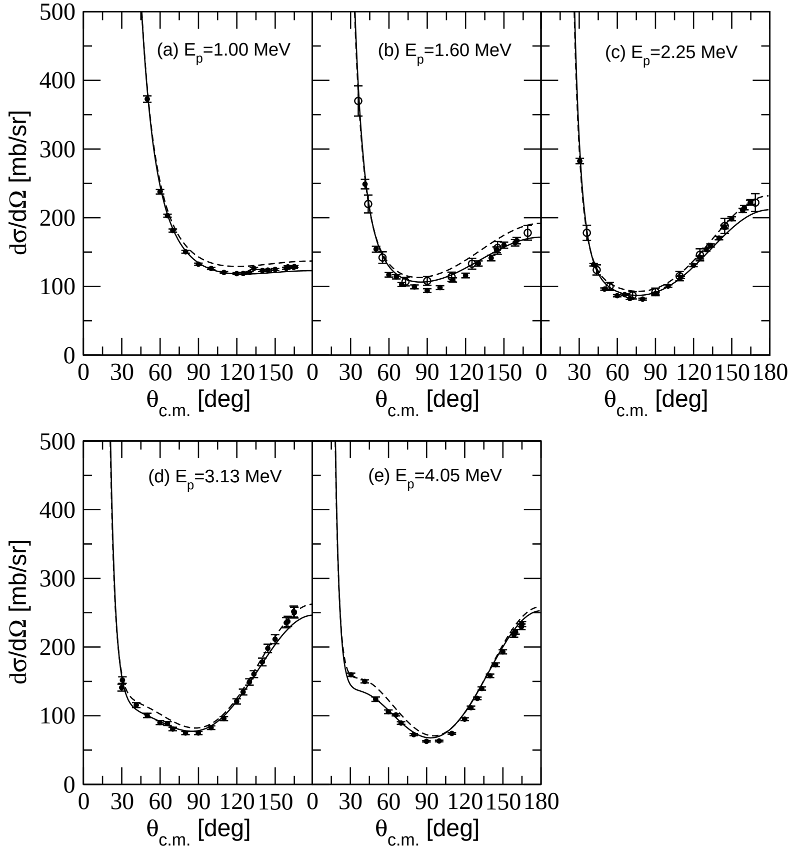
$n -^3$ H total cross section



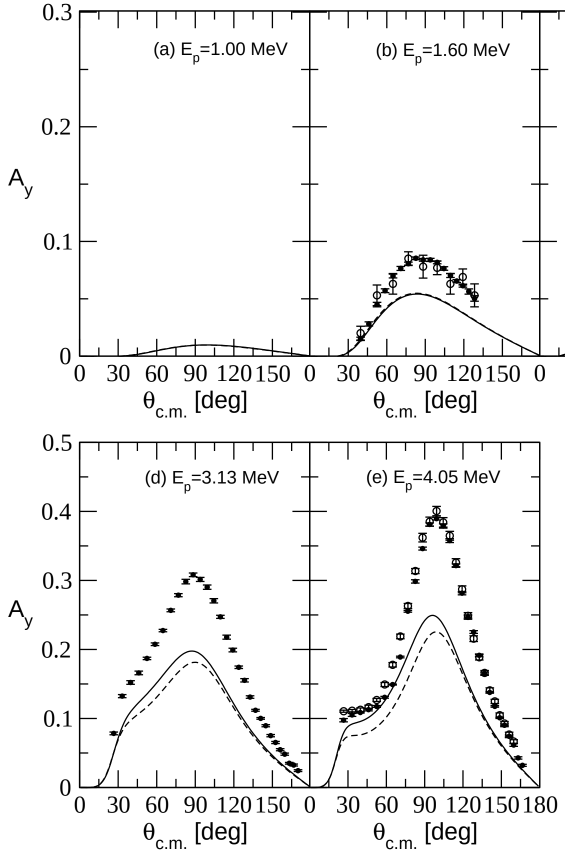
Lazauskas et al. PRC 71, 034004 (2005)

K_1	K_2	K_3	K_4	K_5	K_6	K_7	η	δ (deg)
21							1.00032	10.649
41							1.00107	11.882
61							1.00146	12.136
61	21						1.00131	12.897
61	31						1.00134	13.020
61	37						1.00136	13.055
61	37	21					1.00049	15.923
61	37	31					1.00048	16.105
61	37	35					1.00048	16.132
61	37	35	21				1.00040	16.646
61	37	35	31				1.00040	16.794
61	37	35	31	19			1.00000	17.191
61	37	35	31	19	11	11	1.00000	16.219

Convergence of 0^- inelasticity parameter and phase-shift at $E_p = 4.05$ MeV for $p - {}^3$He scattering. The S-matrix is $S = \eta \exp(2i\delta)$. The potential is AV18.



TUNL data for $p-^3$ He elastic differential cross sections are compared to AV18 (dashed lines) and AV18/UIX (solid lines)



The measured $p-^3$He proton analyzing power. The solid circles are the data from TUNL, open circles from Madison. Calculations are for the AV18 (dashed lines) and AV18/UIX (solid lines)

Conclusions

• Few-nucleon systems are "theoretical laboratories" in which we can test the nuclear interaction

• Benchmarks are used to establish methods:

A=3,4 bound states

N-d scattering and $n -^3$ H scattering

• Detailed wave functions have been constructed using the HH basis for bound and scattering states in $A = 3, 4$

• The HH basis can be used in CS or in MS

• In the low energy sector several observables are well described

• Extension of the HH technique to $A > 4$