On the quantization rules for meson–soliton bound systems

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Received 27 December 1990

The quantization rules for meson–soliton bound systems are discussed within the framework of the bound state approach to strange and heavy flavored baryons. It is shown that the soliton has to change its quantization according to the number of mesons that are bound to it. As a consequence of this, the quantization rule and the quantum numbers of the baryons in the bound state approach are uniquely determined.

Recently, it was shown that the approach to strange baryons originally suggested by Callan and Klebanov [1] provides a very satisfactory description of the masses [2–5] and magnetic moments [6] of hyperons. Moreover, it has been found that this scheme can be extended to the study of charmed and even bottom baryons [7]. The basic assumption in this approach is that while the vacuum is SU(3) symmetric, the fluctuations around the soliton background in the isospin direction and the strangeness direction are to be treated on a different footing, namely as rotational modes for isospin and vibrational modes for strangeness. In this scheme low-lying hyperons appear as bound states of kaons in an SU(2) soliton background. More than one kaon can be put in such a bound state. In this way, systems of one bound kaon correspond to the Λ, Σ and Σ* particles, two kaons give Ξ and Ξ* and three kaons Ω−. It is expected that the addition of further kaons will lead to unbound states [8]. One can find the mass splittings between particles with the same value of strangeness when one considers the SU(2) rotational modes. There is however an important rule that one has to follow in order to obtain the correct quantum numbers of the particles mentioned above: for odd values of strangeness (\( S = -1, -3 \)) the rotor wave function should take only integer values of isospin (spin) while for even values of strangeness (\( S = 0, -2 \)) it should take half-integer values. We have therefore the remarkable feature that the way in which the rotor should be quantized depends on the number of kaons that are bound to the soliton. As will become clear later, this rule also guarantees that the hyperons behave as fermions within this approach. Up to now this rule has been used without further support apart from the fact that it gives the correct quantum numbers of the physical hyperons. In this article, we will argue that in fact these results can be considered as a natural consequence of the ansatz used for the chiral field and the structure of the chiral lagrangian supplemented by the Wess–Zumino term.

In general the chiral effective action can be expressed in terms of the chiral field \( U(x) \) in the following form:

\[
\Gamma = \Gamma_2 + \ldots + \Gamma_{w2} + \Gamma_{SB},
\]

where

\[
\Gamma_2 = \frac{f_\pi^2}{4} \int \frac{\mathcal{M}_4}{\text{Tr} [\partial_\mu U \partial^\mu U^\dagger]},
\]

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\]
\[ I_{\omega z} = -\frac{1\text{N}_c}{240\pi^2} \int_{M_{4} \times \{0,1\}} \text{Tr}[(U^* dU)^5] \]

with \( M_4 \) denoting (3+1)-dimensional space-time. The dots in eq. (1) stand for higher derivative terms and/or terms including vector mesons. Although those terms are important for soliton stabilization they will not be relevant in our discussion. \( F_{\text{SB}} \) is responsible for the explicit breaking of chiral symmetry.

Following ref [1] we assume that this symmetry is badly broken along the strangeness direction by the mass of the kaon.

The Callan–Klebanov ansatz for the chiral field can be expressed in the following way:

\[ U_{cK} = \begin{pmatrix} A(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} N & 0 \\ 0 & 1 \end{pmatrix} \]

where

\[ N = \exp\left[ i \frac{1}{2} r \cdot \hat{r} F(r) \right], \]

\[ U_{A} = \exp\left[ i \frac{1}{2} \frac{\sqrt{2}}{\pi} \begin{pmatrix} 0 & K \\ K^* & 0 \end{pmatrix} \right], \]

\[ K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \]

In eq (2) the matrix \( A \) represents the collective coordinates associated with the quantization of the isospin modes, \( N \) is the square root of the Skyrme hedgehog ansatz, \( F(r) \) is the corresponding radial profile function, and the matrix \( U_{cK} \) belongs to \( SU(3) \) of course.

In order to test whether a given state behaves as a boson or as a fermion we will follow the method used by Witten [9]. Namely, we will compare the amplitude for two processes: In one process, the meson-soliton bound system remains at rest for a long time \( T \). The amplitude is \( \exp(-iMT) \), \( M \) being the mass of the system. In the second process, the system is adiabatically rotated through a \( 2\pi \) angle in the course of a long time \( T \). To investigate what happens in this second case we need to know how the collective variables \( A \) and the kaon field \( K \) transform under a spatial rotation. It was shown in ref. [1] that the effect of such a transformation is (for simplicity we choose a rotation along the \( z \)-axis)

\[ A \rightarrow A \exp(-i\tau_r/2), \]

\[ K \rightarrow \exp(i\alpha A_3) K, \]

Here, \( \tau_r/2 \) is the projection of the isospin operator along the \( z \)-axis and \( A_3 \) is the corresponding projection of the grand-spin operator \( A \) (= \( \tau/2 + L \)).

It is clear that after a \( 2\pi \)-rotation, \( A \) goes to \(-A\), while \( K \) goes to \( \pm K \) depending on the total grand-spin of the \( K \) field. Furthermore one can easily check that eq. (2) is invariant under the combined discrete transformations \( A \rightarrow -A \) and \( K \rightarrow K \). If one now restricts the collective variables \( A \) to \( SU(2) \), one encounters a problem: there is no continuous symmetry of the ansatz (2) that allows a transformation from \(-A\) back to \(+A\) and that belongs to \( SU(2) \). This leads to the well-known fermion–boson quantization ambiguity of \( SU(2) \) skyrmions [10]. Because of the discreteness of the symmetry group of the ansatz \( U \), the physical states could either transform as even or odd functions of \( A \) (i.e., they belong to either one of the two inequivalent representations of the group \( \mathbb{Z}_2 \)). In the case of the normal \( SU(2) \) soliton, the former ones have integer angular momentum and isospin and transform as bosons, the latter ones have half-integer angular momentum and isospin and transform as fermions. In the Callan–Klebanov case this assignment is the same for integer grand-spin of the \( K \) field. For half-integer grand-spin, however, the representations with integer isospin have to transform as fermions and the ones with half-integer isospin as bosons, since the additional phase from the grand-spin of the kaon field inverts the overall sign of the total wave function under a spatial \( 2\pi \) rotation.

In the case of the normal \( SU(2) \) skyrmion, Witten suggested that the mentioned ambiguity could be avoided by assuming the existence of a third light flavor and therefore embedding the \( SU(2) \) group of collective coordinates in \( SU(3) \). Due to the particular way in which the \( SU(2) \) skyrmion is embedded, the right hypercharge transformation \( Y_R \) is a symmetry of the ansatz. This transformation allows us to undo the \( 2\pi \) spatial rotation. However, Witten has shown that in this process one gets a phase \((-)^{N_{c}\chi}\) from the Wess–Zumino term which implies that the resulting baryon (with baryon number \( B=1 \)) be-
haves as a fermion if \( N_c \) is odd. Moreover, the existence of the \( Y_R \)-symmetry gives a constraint on the isospin/spin quantum numbers of the rotor wave function \([11,12]\). The basic idea of the present work is to repeat this kind of construction for the case of the Callan-Klebanov ansatz. Namely, we will assume the existence of a third light flavor — degenerate with the u- and d-flavor — (that we will call “funny strange” flavor) to which in principle we will not assign any physical meaning. Later on we will demand our physical states not to contain any “funny strange” component. A possible alternative interpretation of the “funny strange” flavor will be discussed later.

We embed the CK ansatz in \( SU(4) \) in the following way:

\[
U_{\text{CK}} = \begin{pmatrix} A(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} N & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A^\dagger(t) \\ 0 & 1 \end{pmatrix}
\]

\( (5) \)

Now, \( A \) belongs to \( SU(3)^{(f)} \) (where we introduce here and in the following the label \((f)\) in order to distinguish this group from the original \( SU(3) \)) and \( U_k \) has the same form as before except that

\[
K = \begin{pmatrix} K^+ \\ K^0 \\ 0 \end{pmatrix} .
\]

It is easy to check that this extended \( U_{\text{CK}} \) is invariant under the following time-dependent transformation:

\[
A \rightarrow A \exp \left[ i\beta(t)Y^{(f)} \right] \tag{6}
\]

and

\[
K \rightarrow \exp \left[ -i\beta(t)/3 \right] K , \tag{7}
\]

where

\[
Y^{(f)} = \frac{i}{2} \text{diag}(1, 1, -2) .
\]

At this point we already notice an important difference with respect to the \( SU(3) \) hedgehog \([11]\). In that case, the ansatz was invariant under transformation (6) alone. In our case, we need to perform transformation (7) simultaneously in order to keep the chiral field invariant. It is therefore clear that when transformation (6) is used to undo the \( 2\pi \) spatial rotation on the collective coordinates we will pick up a phase not only from the piece of the Wess-Zumino term depending on the collective coordinates but also a phase coming from the kaon lagrangian. The explicit way to see how this mechanism works is to replace the ansatz (5) in the effective lagrangian. Since the ansatz (5) has the general form \( U_{\text{CK}}(r, t) = A(t)U_0(r, t)A^\dagger(t) \), the Wess-Zumino action separates into (see ref. \([12]\))

\[
\Gamma_{\text{WZ}}(U_{\text{CK}}) = \Gamma_{\text{WZ}}(U_0) + \frac{N_c}{48\pi^2} \int \text{Tr} \left[ (A^\dagger \dot{A} + U_0^\dagger A^\dagger U_0)(U_0^\dagger dU_0)^3 \right] .
\]

Expanding now this term and the non-anomalous piece of the total action up to second order in the kaon field and keeping contributions up to \( O(N_c^{-1}) \), one can find in a straightforward algebraic calculation the following result for the effective lagrangian:

\[
L = L_0 + L_K + L_{\text{rot}} + \frac{1}{3} \sum_i \left( \text{Tr} \left[ A^\dagger \dot{A} \lambda_i^{(f)} \right] Q_i \right) + \frac{1}{4} \text{Tr} \left[ A^\dagger \dot{A} Y^{(f)} \right] (N_c B + \mathcal{S}) . \tag{8}
\]

Here, \( L_0 \) represents the classical lagrangian of the soliton \( (= -M_{\text{sol}}) \). \( L_K \) is the lagrangian of the kaon field to \( O(N_c^0) \) which has the general structure

\[
L_K = \int d^3r[f(r)\dot{K}^\dagger K + \dot{g}(r) (\dot{K}^\dagger K - K^\dagger \dot{K}) + K^\dagger \dot{V} K] . \tag{9}
\]

The radial function \( g(r) \) is associated with the Wess-Zumino term and \( \dot{V} \) is an operator that does not contain any time derivative. \( Q_i \) is responsible for the hyperfine splittings between the hyperons and it is in general a volume integral of a rather involved function of \( K \), its conjugate momentum \( P \) \( (= -\delta L_K/\delta \dot{K}^\dagger) \) and the soliton backgrounds. Typical expressions for \( Q_i \), the radial functions \( f(r) \), \( g(r) \) and the operator \( \dot{V} \) can be found in refs. \([2-4]\) \( \mathcal{S} \) is the strangeness operator. Its explicit form (see also ref. \([3]\)) can easily be obtained by using the Noether construction on eq. (9). One finds that the conserved charge corresponding to the strangeness transformation

\[
K \rightarrow \exp(-i\mathcal{S})K \tag{10}
\]
Finally, $B$ is the baryon number and $L_{\text{rot}}$ is associated with the kinetic energy of the rotating soliton

$$L_{\text{rot}} = 2\theta_1 (\phi_1^2 + \phi_2^2 + \phi_3^2) + 2\theta_2 (\phi_1^2 + \phi_2^2 + \phi_3^2) + 2\theta_3 (\phi_1^2 + \phi_2^2 + \phi_3^2),$$

where we have used $a = 1, 2, 3$.

In eq. (12) $\theta_1$ and $\theta_2$ represent moments of inertia. Their explicit expressions in terms of the soliton backgrounds depend upon the explicit form of the effective action eq. (1).

A very important remark is that there is no kinetic piece associated with the rotation along the "8th-axis". The velocity $\dot{\phi}_8$ along this axis appears only linearly in the last term of eq. (8). The fact that there is no quadratic piece in $\dot{\phi}_8$ in the lagrangian implies that there is a constraint on the possible values that the "funny" right hypercharge $Y^{(r)}_R$ can take [11,12]. According with the standard quantization techniques of constrained systems this constraint is given here by

$$Y_R^{(r)} = \frac{N_c B + \mathcal{S}}{3}.$$

Eq. (13) looks similar to the one obtained in refs. [11,12] except that in addition to the piece proportional to the baryon number, there is here an extra piece $\mathcal{S}/3$.

We can now discuss the fermionic behaviour of the hyperon wave functions. Note that a $2\pi$-spatial rotation of the complete system (5) implies a $-2\pi$-isospin rotation on the collective coordinates (see eq. (3)) and a $2\pi$ rotation in the grand-spin acting on the kaon field (given by eq (4)). The $-2\pi$-isospin rotation on the collective coordinates can be undone by a $3\pi$-hypercharge rotation (see eq. (6)). Inserting this in the lagrangian (8) we obtain the following change in the corresponding action.

$$(\delta\Gamma)^{\text{rot}} = (N_c B + \mathcal{S})\pi.$$

At the same time we should take care of the piece of

the $2\pi$-spatial rotation acting on the kaon field, eq. (4). Each bound kaon carries one (negative) unit of strangeness and a half-integer value of $I$, since it is when bound to an SU(2) hedgehog – an eigenstate with respect to strangeness and grand spin. Therefore a $2\pi$ grand-spin transformation acting on bound kaons can be regarded as a $-\pi$-strangeness rotation (see eq. (10)). The corresponding change in the action is

$$(\delta\Gamma)^{\text{rot}} = -\mathcal{S}\pi.$$

Therefore, the amplitude corresponding to the process in which the meson–soliton bound system is adiabatically rotated by an angle $2\pi$ is

$$\exp\{i((\delta\Gamma)^{\text{rot}} + (\delta\Gamma)^{\text{rot}})\} \exp(-\mathcal{S}\pi) = (-)^{N_c B} \exp(-\mathcal{S}\pi).$$

In this way we prove that in our enlarged flavor space the $B=1$ meson–soliton bound system must be quantized as a fermion if $N_c$ is odd (as it was the case for the SU(3) hedgehog [11,12]).

We will finally show that the extra piece in eq. (13) leads to the quantization rules for the rotor wave function mentioned at the beginning of the present article. In general any SU(3) representation can be spanned in terms of $p$ $3$ and $q$ $\overline{3}$ fundamental representations. We are only interested in minimal SU(3) ($^{(t)}$) representations and in those SU(2) sub-representations which do not contain any "funny strange" component. This means that the rotor states belonging to these sub-representations should have the maximal value of "funny" hypercharge, $Y^{(r)}_R$, of their SU(3) ($^{(t)}$) multiplet and furthermore that $Y^{(r)}_R = Y^{(r)}_R$. The possible values of $Y^{(r)}_R$ and $I_{\text{rot}}$ (the rotor's isospin which is equal to its angular momentum $J_{\text{rot}}$ because of the hedgehog property of the rotor) are given by

$$Y^{(r)}_R = Y^{(r)}_R = \frac{p+2q}{3},$$

$$I_{\text{rot}} = J_{\text{rot}} = \frac{p}{2}.$$

These results follow from the fact that in the $3$ representation the state of maximal $Y^{(r)}_R (=\frac{1}{3})$ has isospin $\frac{1}{2}$, whereas in the $\overline{3}$ representation the state of maximal $Y^{(r)}_R (=\frac{1}{3})$ has isospin $0$.

As shown in ref [1] the isospin $I$ of the hyperon is given by the isospin $I_{\text{rot}}$ of the rotor alone, whereas
the spin $J$ of the hyperon is the sum of the angular momentum $J_{\text{rot}}$ of the rotor and the total grand spin $A$ carried by the kaon field.

If we assume $B=1$ and $N_c=3$, we can extract from eqs. (13), (14) which values of spin/isospin are allowed for different values of strangeness. The results are shown in table 1. In constructing this table we have assumed that only the energetically lowest bound state $(A_{bs}, l_{bs}, S_{bs}) = (\frac{1}{2}, 1, -1)$ is populated. In the case of hyperons with strangeness $S = -2, -3$ this assumption constrains the possible values of the total grand-spin $A$ carried by the kaon field. In fact, as the total kaon wave function should be completely symmetric only the state with maximum $A$ is allowed. Of course this restriction does not hold when not only the lowest bound state $(A_{bs}, l_{bs}, S_{bs})$ is populated or when different kinds of mesons ($K, D, ...$) are bound to the soliton (as it is the case when strangeness and charm excitations are considered simultaneously).

As we see in table 1 the values of isospin/spin that the rotor wave function can take (and which also determine the isospin of the hyperon itself) depend on the number of kaons bound to the soliton. In particular they should be integer or half-integer depending on whether $S$ is odd or even respectively. Moreover, the SU(3)($^f$) representations which characterize the rotor wave function (see table 1) correspond in the quark model language to those minimal multiplets which only involve the $u$ and $d$ quarks of a given hyperon, e.g. in the case of the cascades the SU(3)($^f$) representation of the rotor is a triplet, for the $\Omega^-$ it is a singlet. In addition kaons when bound to the soliton take over the role of strange quarks. Thus, there is no non-strange component in the $\Omega^-$ within the bound state approach, although it is formed by three kaons bound to a hedgehog background. This settles the problem about the structure of the $\Omega^-$ raised in ref. [7].

So far, we have discussed the case of strange baryons. However, all our arguments can also be applied to baryons with charm ($c$), bottom ($b$) and even top ($T$) quantum numbers. Eq. (13) can easily be generalized to include different heavy flavors:

$$Y_k^{(f)} = \frac{N_c B + S - \ell + b - T}{3}.$$  \hspace{1cm} (15)

Note that when the model is extended to include charm or bottom, one could assign a physical interpretation to the "funny strangeness". In fact, since strange excitations are light in comparison to the charm and bottom ones, one might be tempted to omit the word "funny" and treat the strangeness on the same footing as the light $u$ and $d$ flavors \#1. Then, however, the strangeness modes – see the terms involving $\Theta_2$ in eq. (12) – are to be quantized as rotational modes like the isospin ones. Although this approach is conceptually nice and simple, it is known to fail phenomenologically [13]. Therefore we prefer to be consistent within the bound state approach and consider the "funny strange" flavor just as a mathematical trick for avoiding the problem of the SU(2) quantization ambiguity.

An alternative to the extension of $A$ to SU(3)($^f$) as done in eq. (5) could be an extension to SU(2)×U(1)$_Y$. In such a case one would have found a simi-

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**Table 1**

Predicted values of isospin $I$ and spin $J$ of the $B=1$ hyperons with strangeness $S$. Within the bound-state approach these values are extracted from eqs. (13), (14) for the case of three colors, $N_c=3$. Furthermore, the corresponding values of the total grand spin $A$ (carried by the kaon field) and of the rotor's "funny" right hypercharge $Y_k^{(f)}$ are reported and the SU(3)($^f$) representations of the rotor including their labels $p$ and $q$ are given. Finally, the identified physical particles are listed.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$Y_k^{(f)}$</th>
<th>$p$</th>
<th>$q$</th>
<th>$I=J_{\text{rot}}$</th>
<th>$A$</th>
<th>$J$</th>
<th>Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>10</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
</tr>
<tr>
<td>-1</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\Lambda$</td>
</tr>
<tr>
<td>-1</td>
<td>$\frac{3}{2}$</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>$\Sigma, \Sigma^*$</td>
</tr>
<tr>
<td>-2</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\Omega$</td>
</tr>
</tbody>
</table>
lar expression to eq. (8) with the difference that the last summand would read \((1/2)\text{Tr}(A^t A Y) (N_c B + 3/2)\). The fermion/boson quantization rule would therefore be the same in both cases. However, there would not follow a rule for the quantum numbers, since the constraint on the right hypercharge is decoupled from the SU(2) representations of the rotor wave function because of the product structure of SU(2) \times U(1).

In conclusion, we have studied in detail the quantization rules for the meson–soliton bound systems within the bound state approach to strange and heavy flavored baryons. It was discussed that when only two light flavors are considered there is an ambiguity on whether the system should be quantized as a fermion or as a boson. However, it was shown that when a third light flavor is added the quantization rules for the system and the quantum numbers of the baryons are uniquely determined. For odd values of \(N_c (N_c = 3\) in nature) baryons – as described in this approach – behave as fermions. Moreover, the quantum numbers obtained for the low-lying physical baryons coincide with those given by the quark model and no spurious state is predicted. There is the very interesting possibility that some of the results obtained in the present article can be interpreted from a geometrical point of view [14]. Namely that the extra phase induced by the kaon fields when the collective coordinates are rotated by a given angle can be regarded as a Berry phase [15]. Work along this line is being carried out.

We are grateful for helpful discussions with M A. Nowak and M Rho.

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